

Opportunity I

You're awesome! First of all, you've gotten through a semester of Calculus. Secondly, you've all been working very hard this semester. This is your first chance, your first Opportunity, to show off what you've learned with all of that hard work. Good luck!

There are no calculators and no cell phones allowed. You may not copy someone else's work. Show your work wherever possible - partial credit will be given where appropriate.

1. In each of the following, find y' .

$$y = \ln(e^{x^2})$$

$$y = xe^x$$

$$y = \ln \frac{(2x+1)^3}{(3x-1)^4}$$

$$\sin(xy) = e^{\sqrt{\pi}x} + \ln y$$

$$y = \ln(x + \sin(x))$$

$$y = x \int_2^x \frac{1}{t} dt$$

2. Integrate:

$$\int \frac{x}{3-x^2} dx$$

$$\int \frac{3-x^2}{x} dx$$

$$\int (x+1)e^{(x^2+2x-3)} dx$$

$$\int_{\ln 2}^{\ln 3} \frac{e^x}{e^x-1} dx$$

3. Is the function $h(x) = (e^{2x} + 1)$ one-to-one or not? Explain.

Let $f(x) = \ln((1+x)^3)$.

What is the domain of f ?

Find $f^{-1}(x)$.

Suppose $g(3) = 1$, g and h are inverse functions, and $h'(1) = 5$. What can you say about g' ?

4. Find the derivative of $\arccos x$. Show your work.

For the following function, find y' .

$$y = \sin(\arccos(x))$$

5. Let $f(x) = x^2$ and $g(x) = 3 - 2x$.

a. Find the finite area between f and g .

b. Suppose the area in part a. is rotated around the x -axis. Write an integral which gives the volume of the resulting solid. You do not need to integrate.

c. Suppose the area in part a. is rotated around the line $x = 2$. Write an integral which gives the volume of the resulting solid. You do not need to integrate.

6. Let A be the region bounded between the function $y = x^2$, the x -axis, and the line $x = 1$. Will the following formula correctly calculate the volume of the solid formed when A is rotated around the x -axis? If you think it is correct, explain why it works. If you think it isn't, correct the formula (with as few changes as possible).

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \pi \left(\frac{k}{n}\right)^2 \left(\frac{1}{n}\right).$$

Extra Credit: "Freude, schoene Goetterfunken, Tochter aus Elysium, wir betreten feuertrunken, Himmlische, dein Heiligtum!" (Translation from German: What happiness! Your beautiful divine sparks, daughters from the Elysian fields; we present, drunk with fire, heavenly, your holiness!)

For one point each, name the poet who wrote these words, the composer who used them, and the piece in which they were used.