

## Final Opportunity

1. On the set of axes below, sketch the direction field for the differential equation  $y' = xy^2$ .

On the same axes, draw the solutions which pass through the points  $(1, 2)$  and  $(2, 0)$ .

Solve the initial value problem  $y' = xy^2$ ,  $y(1) = 2$ .

2. (5 pts each)

Solve the following integrals

a)  $\int x \cos x \, dx$

b)  $\int \frac{\ln x}{x} \, dx$

c)  $\int_0^\infty \frac{\pi}{9+x^2} \, dx$

d)  $\int \frac{1}{x^2-1} \, dx$

e)  $\int_0^\infty \frac{1}{\sqrt{x}} \, dx$

What's your major and why?

3. North Korea may have produced as much as 100 kg of Plutonium-239 which has a half-life of approximately 24,000 years. If they stop making it now, how many years before their Pl-239 stash decays to 90 kg? (If you'd like to check the plausibility of your answer with a calculator's approximation, ask Dave.)

4. Prove that the series  $\sum_{n=1}^{\infty} \frac{1}{n}$  diverges. You may not simply cite the p-test.

5. Find the third Taylor polynomial  $T_3(x)$  for the function  $f(x) = \ln(1+x^2)$  near  $x = 0$ . (For 3 bonus points, find the entire Taylor series, using summation notation to express it.)

6. Having completed a year of Calculus, you should not be able to state both parts of

### The Fundamental Theorem of Calculus.

Do so.

Part I.

Part II.

7. For each of the following, determine convergence or divergence. If the series converges, state whether it converges absolutely or conditionally.

a)  $\sum_{n=1}^{\infty} n^{-e}$

b)  $\sum_{n=1}^{\infty} \frac{2^n + 4^n}{5^n}$

c)  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{1/n}$

d)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}(n+1)^{1/2}}$

- e) For what values of  $x$  does the series  $\sum_{n=1}^{\infty} \frac{x^n}{(n+1)2^n}$  converge? Include an analysis of what happens at the endpoints.

**Extra Credit:** What does SARS stand for? How likely is it that someone who catches SARS will die?