Calculus II Name:____________________

Opportunity II: Spring Break's Revenge

Here you go! This is your second chance to show me what you've learned this semester. No calculators are allowed. If you have any questions, please ask me. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. (10 pts) State the Test for Divergence.

What does this test tell you about the series $\sum_{k=1}^{\infty}(-1)^k$? Explain.

What does this test tell you about the series $\sum_{k=1}^{\infty}\frac{(-1)^k}{k}$? Explain.

2. (15 pts.) Find the sum of the following series. Justify your answers.

$$\sum_{n=1}^{\infty}\frac{2^n+4^n}{5^n}$$

$$\sum_{n=1}^{\infty}\frac{2}{(2n+1)(2n-1)}$$

3. (5 pts each) Do the following sequences converge or diverge? If they converge, what do they converge to? Explain your answers.

$a_n = 2 + (-1)^n$

$b_n = \frac{\pi^2}{6} + \frac{1}{n}$

$c_n = \frac{\sin n}{\sqrt{n}}$

$d_n = \frac{\sqrt{n}}{\ln n}$

4. (10 pts) **The Integral Test.** Use the following picture to fill in the bounds on the sums in the inequalities which one uses to prove the Integral Test. Draw boxes on the graph which explain your answer and give a brief written explanation as well.

$$\sum_{n=}\ g(n) \quad \le \int_4^{\infty} g(x)\,dx \le \sum_{n=}\ g(n)$$

5. (15 pts) Suppose the series $\sum_{n=1}^{\infty} a_n$ converges to 10.

What happens if we change a_{103} from its original value of $\frac{1}{1000}$ to $100,000$? Does the new series converge or diverge? Why?

What happens if, going back to the original series, we change every 100^{th} term to 1, so that $a_{100} = 1$, $a_{200} = 1$, $a_{300} = 1, \ldots$? Does the series converge or diverge? Why?

6. (30 pts) For each of the following, determine convergence or divergence. Justify your answers.

a) $\sum_{n=1}^{\infty} n^{-3}$

b) $\sum_{n=1}^{\infty}\frac{\pi^n}{2^n+4^n}$

c) $\sum_{n=1}^{\infty} ne^{-n}$

d) $\sum_{n=1}^{\infty}\frac{n^3}{n(n+1)(2n-1)}$

e) $\sum_{n=2}^{\infty} \frac{1}{n(n^2-1)^{1/3}}$

f) $\int_0^{\infty} \frac{1}{1+x^2}\, dx$

Extra Credit: After many months of diplomatic efforts, the U.S. has been joined by a "coalition of the willing" in its war against Iraq. Name all of the other countries which have contributed forces to this coalition. (One point each. For comparison, 17 countries contributed troops to the first Gulf War.)