Calculus II Exam I Feb. 6th, 2002

This is an un-timed exam. You may use a calculator which doesn't perform symbolic differentiation and integration as long as you have not programmed relevant information into it. (Yes, I reserve the right to check.) If you don't have a calculator, but would like a computation done, raise your hand and I will assist you. Scratch paper will be available - this will be especially appropriate for the integrals.

1. (55 pts - 5 pts each) Compute the following integrals:

$\int_{-1}^{1} \frac{x}{1+x^2}\,dx$

$\int \frac{x^2}{1+x^2}\,dx$

$\int \sin x\sqrt{\cos x}\,dx$

$\int 97\pi x + \frac{\sqrt{3}}{x} - \frac{1}{x^2}\,dx$

$\int \frac{3}{\pi + x}$

$\int t\sqrt{1+t^2}\,dt$

$\int ze^{z^2}\,dz$

$\int xe^x\,dx$

$\int \sin xe^x\,dx$

$\int \ln x\,dx$

$\int \arcsin x\,dx$

[Draw a picture here.]

2. (15 pts) What is the connection between the product rule for derivatives and the method of integration called "Integration By Parts"?

3. (10 pts) Ten years after the tragic accident at the Chernobyl nuclear power plant, investigators were finally let into the plant. Among other things, they found 100 kg of a highly radioactive isotope of Plutonium, which has a half-life of just 15 years. How much of this isotope were in the plant when the accident happened? (Hint: Let $t = 0$ when the investigators came in, and find out how much Plutonium was present at $t = -10$.)

4. (10 pts) Solve the differential equation

$$y''(t) = -4y(t),$$

with the initial conditions $y(0) = 2, \quad y'(0) = -4$.

5. (10 pts) Show that the function

$$y(t) = C_1e^t + C_2e^{-t}$$

solves the differential equation

$$y''(t) = 4y(t).$$

Give the solution to $y''(t) = 4y(t)$ under the initial conditions $y(0) = 1, \quad y'(0) = 6$ (i.e. solve for the constants C_1 and C_2 in this case.)

Extra Credit: Name up to five democratic countries which have a majority Muslim population.