Calculus II Name:_____

The Big Opportunity

Here it is, your final opportunity to show off the wonderful knowledge of Calculus that you've gained over the past year. As always, explain all of your answers.

1. State both parts of the Fundamental Theorem of Calculus (FTC).

Use one part of the FTC, along with the definition of the function $\ln x$, to show that $(\ln x)' = \frac{1}{x}$.

Use the other part to solve the integral $\int x^2 + 3 \, dx$.

2. Let A be the region in the first quadrant bounded between the functions $y = x^2 + 2$ and $y = 3x^2$.

Suppose A is spun around the y-axis. Write an integral (but do not solve it) which would calculate the volume of the resulting solid.

Suppose A is spun around the x-axis. Write an integral (but do not solve it) which would calculate the volume of the resulting solid.

3. Do the following integrals. If an integral is improper, write it out with limits.

4. For each series below (and on the following pages), determine if it converges or diverges. Be sure to indicate any tests you are using. If a series converges and it's possible to find the sum, do so (this is true of two of the series). If a series converges, but only conditionally (and not absolutely), say why.

$\sum_{k=1}^{\infty} \frac{1}{k!}$ (extra credit: What does this series converge to?)

$\sum_{k=1}^{\infty} \frac{k \, 2^k}{e^k}$

$\sum_{k=2}^{\infty} \frac{\ln k}{k+4}$

$\sum_{k=1}^{\infty} \frac{(-1)^k}{(1+\frac{1}{k})^k}$

$\sum_{k=1}^{\infty} \frac{5k + 97\pi}{\sqrt{3k^3 + 2k + 4}}$

5. Find the 4th degree Taylor polynomial near 0 for the function $f(x) = \cosh x = \frac{e^x + e^{-x}}{2}$.

6. Write a problem which you think should have been on this exam. Write a solution as well. You will be graded not only on your work, but on how appropriate the problem is (i.e. $2 + 2 = ?$ will get you 0 points).