Calculus II Name:____________________

Opportunity III: Take It Home

Rules: This is a take-home Opportunity. You may not consult anyone about these particular problems. You may talk to *anyone* about similar problems, and you may consult any books you want. You can even use your calculator if you want. However, everything you hand in must be your own work. If I don't believe that it is your work, I reserve the right to make you explain your reasoning to me in person. On the following pages, place your final answers (thank you, Regis). All scratch work should be done on separate sheets (and not handed in). This Opportunity is done on Wednesday, April 25th at 10:30 am. If you think there is a typo, email me immediately.

1. Use the definition of the limit of a sequence (involving ϵ and N) to prove that $\lim_{n\to\infty} \frac{1}{n^{3/2}} = 0$. (Hint: The first line of your proof should read "Take $\epsilon > 0$ and choose $N = \ldots$".)

2. Prove that $\lim_{n\to\infty} \frac{\ln 5n}{n} = 0$.

Prove that $\lim_{n\to\infty} (5n)^{1/n} = 1$.

3. Determine if the following integrals converge or diverge. If they converge, calculate the value they converge to.

$$\int_0^\infty \frac{1}{1+x^2}\,dx$$

$$\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$$

4. For each series below (and on the following pages), determine if it converges or diverges. Be sure to indicate any tests you are using. If a series converges and it's possible to find the sum, do so (this is true of two of the series). If a series converges, but only conditionally (and not absolutely), say why.

$$\sum_{k=1}^\infty \frac{1}{k^2+3k}$$

$$\sum_{k=1}^\infty \frac{\ln k}{\sqrt{k+1}}$$

$$\sum_{k=1}^\infty \frac{1}{1+k^2}$$

$$\sum_{k=1}^\infty \frac{k^k}{e^k k!}$$

$$\sum_{k=1}^\infty \frac{k!}{k^k}$$

$$\sum_{k=1}^\infty \frac{194(\pi-2)^k}{\pi^k}$$

$$\sum_{k=1}^\infty \frac{\sin k}{\sqrt{k}}$$

$$\sum_{k=1}^\infty \frac{(-1)^k}{\ln k}$$

$$\sum_{k=1}^\infty \frac{1}{\sqrt{3k^3+2k-4}}$$

5. For what values of x does the series

$$\sum_{k=1}^{\infty} \frac{k! x^k}{k^k}$$

converge absolutely? (Note that since x might be negative, the series might be alternating.)

For what values of x does it (the same series) diverge?

For what values of x does it converge conditionally?