Calculus II   Name:_____

Opportunity II

Here you go! You're opportunity to show me how hard you've been working, how much you've learned, how good you are at Calculus. As before, no calculators are allowed.

1. (20 pts) For each of the following, find y'.

$y = \ln(x^2 + \sin x)$

$y = e^{\sqrt{x}\tan x}$

$y = \arccos x$

$y = (97\pi)^x$ (If you don't like the number 97π, use $y = 300^x$ instead.)

2. (40 pts) Solve each of the following integrals. Show your work. (If you forget every +C, you will lose points.)

$$\int \frac{dx}{1 + x^2}$$

$$\int \frac{dx}{x^2 - 1}$$

$$\int \frac{x^2}{\sqrt{1 - x^2}}\, dx$$

$$\int \frac{x^2}{1 + x^2}\, dx$$

2. (continued) Solve these integrals

$$\int x^2 e^x\, dx$$

$$\int \sin^2 x \cos^3 x\, dx$$

$$\int \arccos x\, dx$$

$$\int x \cos x\, dx$$

3. (15 pts) This question is in four parts. Follow the directions for each part carefully.

How do we define the function $f(x) = \ln x$? Give a mathematical answer (a formula) as well as a brief description (a sentence or two). Pictures are encouraged.

How do we define the number e? A brief sentence will suffice.

How do we define the function $g(x) = e^x$? Explain your answer.

Graph both $y = \ln x$ and $y = e^x$ below. Label at least 2 points on each curve.

4. (15 pts) Just before noon, a man is found dead in a room with an open vial next to him. The label reads "Dangerous!! Contains 100 g Deathium - begins deadly radioactive decay immediately upon opening!" A detective wearing a protective lead suit finds that at noon, there are 50 grams of the Deathium left in the vial. At 1 pm she re-weighs the Deathium to find that only 40 grams are left.

At what time did the man open the vial of Deathium? (If you feel the need to find an approximate answer, ask Dave to use his calculator to get one.)

5. (10 pts) We've seen that Integration by Parts can be used to solve certain integrals. Explain how the integration by parts formula comes from the product rule. Write in complete sentences, and explain everything clearly.