CALCULUS 152 NAME _____

Opportunity 1

Instructions: Here you go. It's your opportunity to show me how hard you've been working so far. There are no calculators allowed (or needed) on the exam; however, you have as much time as you want (within reason). At all times, you *must* keep in mind that you are a smart person (trust me on this).

Let's start with the more straightforward stuff...

1. For each of the following expressions, find y'.

$y = \ln(x^3 + 1)$

$y = e^{\sin(x^2)}$

$y = e^{x \ln(\sin(x))}$

$y = \arccos x$ (Hint: It may help to write this as $f(x) = \arccos x$, and try to find $f'(x)$.)

$y = \int_3^{x^2} \frac{1}{t}\, dt$

And now for some integrals...

2. Do the following integrals. Remember your $+C$'s where appropriate.

$\int_0^{e-1} \frac{1}{x+1}\, dx$ (simplify your answer)

$\int \frac{1}{(x-1)^2}\, dx$

$\int \frac{e^x}{(e^x+1)^{3/2}}\, dx$

$\int \cot x\, dx$

$\int e^{h(x)} h'(x)\, dx$ where $h(x)$ is a differentiable function.

3. Consider the function $f(x) = e^{-x}$.

Draw a rough sketch of the graph of f; label at least two points.

Find the area under the graph of f between $x = 0$ and $x = b$.

What happens to the area as $b \to \infty$?

4. Let $g(x) = (\ln(x+1))^3$.

Give the domain and range of g.

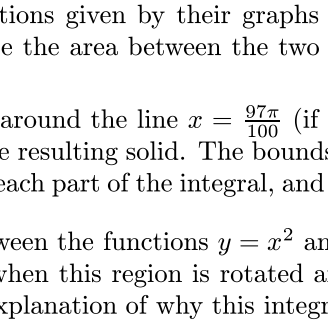
Prove that g is a one-to-one function. (Hint: prove that it is always increasing.)

Find $g^{-1}(x)$.

Give the domain and range of g^{-1}

Washers or Shells?

5. Suppose f g are functions given by their graphs on the right. Let A be the area between the two graphs.



Suppose A is rotated around the line $x = \frac{97\pi}{100}$ (if this bugs you, use the line $x = 3$). Write an integral which gives the volume of the resulting solid. The bounds on the integral should involve the constants a and b. Write a brief explanation of each part of the integral, and an explanation of why this integral gives the correct volume.

6. Sketch the region between the functions $y = x^2$ and $y = \frac{1}{2}x^2 + 2$. Write an integral which gives the volume of the shape generated when this region is rotated around the x-axis. Write a brief explanation of each part of the integral, and an explanation of why this integral gives the correct volume.