

Opportunity III

No calculators or cell phones are allowed. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. Answer some general questions about sequences and series.

a) What's a sequence? Give an example of a convergent one and a divergent one.

b) What's a series? Give an example of a convergent one and a divergent one.

c) Does the sequence $a_n = \frac{n-1}{n^2+1}$ converge or diverge? Explain.

d) Does the sequence $s_n = \sum_{k=1}^n \frac{k-1}{k^2+1}$ converge or diverge? Explain.

2. Completely determine the convergence and divergence of the power series below. That is, determine for which values of x the series converges absolutely, converges conditionally, and diverges. Show your work.

$$\sum_{n=0}^{\infty} \frac{n(x-4)^n}{n^2+1}$$

3. For half credit, prove that this series converges. For full credit, find the sum.

$$\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$$

4. Determine whether the following series converge or diverge. Give reasons for each.

a) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n(n-1)}}$

b) $\sum_{n=1}^{\infty} \frac{n^4+1}{3n^4-1}$

c) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

d) $\sum_{n=1}^{\infty} \frac{1}{n2^n}$

e) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}-1}$

f) What's your major? Why?

5. Assume that $f(x)$ is a positive, decreasing function.

Put these four quantities in order from least to greatest. Draw a picture to illustrate the situation and briefly explain your answer. Be sure to make it clear which area corresponds to which letter.

$$A = \sum_1^{\infty} f(n) \quad B = \int_1^{\infty} f(x) dx \quad C = \sum_2^{100} f(n) \quad D = \sum_2^{\infty} f(n).$$

Which inequality (i.e. $B < D$) shows that "If the integral converges, the sum converges."?

Which inequality shows that "If the integral diverges, the sum diverges."?

6. Find a series that approximates the function $f(x) = \frac{x}{1+x^2}$ near $x = 0$ (i.e. the Maclaurin Series.) Show your work. For full credit, write your answer in summation notation.

Extra Credit: Thursday is Thanksgiving Day. For a point each, name the year and location of the festival generally credited as the first Thanksgiving in the US. (Hint: it didn't involve shopping that day or the next.)