Name: ____________________

Opportunity 2.0 — Fall '12

Yippee! Your first opportunity to improve on your first opportunity!

No calculators or cell phones are allowed — please zip them away in your bookbag. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. Define the following different uses of the *limit*, using a sentence that starts "You can make..."

a) $\lim_{n\to\infty} b_n = x$

b) $\sum_{k=1}^{\infty} a_k = M$

c) $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \text{ the slope of } f \text{ at } x$

d) $\lim_{a\to\infty} \int_0^a f(x)\,dx = L$

2. Integrate each of the following with Integration By Parts (and possibly other methods as well).

a) $\int x\cos(2x)\,dx$

b) $\int x^2 e^x\,dx$

c) $\int e^{\sqrt{x}}\,dx$

d) $\int \text{arctan} x\,dx$

3. Integrate each of the following as a trigonometric integral:

a) $\int \sin^3 x\cos^2 x\,dx$

b) $\int \cos^2 x\,dx$

4. Integrate each of the following with a trigonometric substitution (and possibly other methods as well):

a) $\int \dfrac{x^3}{\sqrt{x^2+9}}; dx$

b) $\int \dfrac{dx}{\sqrt{2-x^2}}\,dx$

5. Integrate the following with partial fractions:

$$\int \frac{2}{x^2+3x-4}\,dx$$

6. Is the area under the function $f(x) = \frac{1}{1+x^2}$ (for all real numbers) finite or infinite? Explain with a computation.

7. What is a series?

What is a sequence?

8. True or False (write the entire word "True" or the entire word "False" after each statement). If False, write a counter example.

a) If $\sum_{k=1}^{\infty} a_k$ converges then a_k converges.

b) If is possible for b_k to converge to 0 and for $\sum_{k=1}^{\infty} b_k$ to diverge.

c) If c_k converges then $\frac{1}{c_k}$ converges.

d) If d_k converges to 0, then $\sum_{k=1}^{\infty} d_k$ converges.

9. For each of the following, determine if it converges or diverges. Explain your reasoning.

$a_n = \frac{1}{n}$

$b_k = \sum_{n=1}^{k} \frac{1}{n}$

$c_n = \frac{\pi 2^n}{5^n}$

$d_n = \frac{\sin n}{\sqrt{(n+1)}}$

$e_n = \frac{\ln 2n}{\ln 5n}$

$f_n : 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1 \ldots$

10. For each of the following, determine if the series coverges or diverges. Then find the sum.

a) $\sum_{n=2}^{\infty} \frac{1}{(n+1)(n-1)}$

b) $\frac{5}{2} + \frac{5}{3} + \frac{10}{9} + \frac{20}{27} + \frac{40}{81} + \ldots$

Extra Credit: Name the 3 largest bodies of fresh water in the world (by surface area, not volume).