Name: _____

# Opportunity 1 — Fall '12

So exciting — your first opportunity to show what you've learned in Calc II!

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

**1.** We'll start with some derivatives. In each of the following, find $y'$ (a.k.a. $\frac{dy}{dx}$).

a) $y = \sqrt[5]{\ln x} = (\ln x)^{\frac{1}{5}}$

b) $y = \cosh(2x)$

c) $y = \sqrt{x}\ln(x)$

d) $y = \int_0^x e^{\sin x}\, dt$

e) $y = 4^x$

f) $y = x^{\tan x}$

The best part of being at St. Mary's (other than the great math professors) is ...

**2.** What is the **definition** of each of the following? (Remember to give the definition we used in class - in sections 7.2* and 7.3*.)

The natural log function $\ln x$ is defined as ....

The number $e \approx 2.71828\ldots$ is defined as ....

The exponential function $\text{Exp}(x)$ is defined as ....

**3.** Use the definition of the natural log function above to prove that $(\ln x)' = \frac{1}{x}$.

**4.** Find the following anti-derivatives:

a) $\displaystyle\int \sqrt{x}(x-1)\, dx$

b) $\displaystyle\int \frac{e^{2x} + e^{-2x}}{2}\, dx$

c) $\displaystyle\int \frac{2x}{x^2+3}\, dx$

d) $\displaystyle\int \pi^x\, dx$

**5.** Let $f(x) = \arctan x$.

Prove that $f'(x) = \frac{1}{1+x^2}$.

Find the derivative of $y = x^2 \arctan x^2$. Make sure your answer depends only on $x$ (and not on $y$).

**6.** A curve and a straight line are shown on the graphs here. One is the graph of $x = y^4 - 1$; the other is the graph of $y = \frac{x}{5} - 1$.

Write an integral which calculates the area between the two.

Calculate the area by doing the above integral.

Write an integral which calculates the volume swept out by rotating the same region *about* the line $y = -1$.

**7.** If you use the method of washers to calculate the volume of a solid of rotation (generated by the region shown), the correct formula is this:

$$\int_a^b \pi[(g(x))^2 - (f(x))^2]\, dx$$

Explain why this formula correctly calculates the volume. Be sure to explain every part of the formula.

**8.** The Fundamental Theorem of Calculus says, roughly, that definite integrals and derivatives undo each other. It's divided into two parts, which can be remembered by thinking of them like this:

- FTC1: The derivative of a definite integral is the function itself.

- FTC2: The definite integral of a derivative is the function itself.

Give a more detailed and precise statement of each part of the FTC:

FTC1:

FTC2:

Go back through the exam and mark every place where you have used the Fundamental Theorem — with "FTC1" where you used the first part and "FTC2" where you used the second part.

*Extra Credit:* For half a point each, name the four people who will appear on ballots in Maryland as candidates for President of the United States.