

## OPPORTUNITY 2.0

FALL '09

Yippee! Your first opportunity to improve on your first opportunity!

No calculators or cell phones are allowed — please zip them away in your bookbag. If you have any questions, please ask Lucia or Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. Let  $A$  be the bounded region between the graphs of  $y = x^3$  and  $y = 4x$  as shown.

- Write an integral that calculates the area of  $A$ .
- Using the method of washers, write an integral that calculates the volume of the region formed when  $A$  is rotated around the  $y$ -axis.
- Using the method of shells, write an integral that calculates the volume of the region formed when  $A$  is rotated around the  $y$ -axis.
- Calculate either b) or c) (or both, if you want to check your answers).

2. Define the following:

a)  $\lim_{n \rightarrow \infty} a_n = L$

b)  $\sum_{k=1}^{\infty} b_k = M$

c)  $\lim_{x \rightarrow 2} f(x) = \infty$

3. (You get a second chance!) Precisely state the Fundamental Theorem of Calculus (both parts). Briefly explain what it means.

4. True or False (write the entire word “True” or the entire word “False” after each statement):

a) If  $\sum_{k=1}^{\infty} a_k$  converges then  $a_k \rightarrow 0$ .

b) If  $a_k \rightarrow 0$  then  $\sum_{k=1}^{\infty} a_k$  converges.

c) If  $a_k$  converges then  $(a_k)^2$  converges.

5. Integrate the following:

a)  $\int \frac{1}{1+x^2} dx$

b)  $\int \frac{x}{1+x^2} dx$

c)  $\int \frac{x^2}{1+x^2} dx$

d)  $\int \frac{1}{1-x^2} dx$

e)  $\int \frac{x}{\sqrt{1+x^2}} dx$

f) Tell me about the happiest day of your life so far.

**6.** More integrals!

a)  $\int \sin x \, dx$

b)  $\int x \sin x \, dx$

c)  $\int \sin^3 x \cos^2 x \, dx$

d)  $\int x \ln x \, dx$

**7.** One last integrals:

$$\int \frac{x^3}{\sqrt{1+x^2}} dx$$

**8.** For each of the following, determine if it converges or diverges. Explain your reasoning.

$$a_n = \frac{1}{n}$$

$$b_n = 2^{-n}$$

$$c_n = \frac{\ln n}{n^3}$$

$$d_n = \frac{\ln n}{\ln(\ln n)}$$

$$e_n : \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, \dots\}$$

**9.** For each of the following, determine if the series converges or diverges. Explain your reasoning.

a)  $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \dots$

b)  $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$

*Extra Credit:* Name the play that contains characters named Bottom, Puck, and Flute. For an additional point, name the playwright.