

OPPORTUNITY 1

FALL '09

Yippee! Your first opportunity to show off what you've learned so far!

No calculators or cell phones are allowed — please zip them away in your bookbag. If you have any questions, please ask Lucia or Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. We'll start with some derivatives. In each of the following, find y' (a.k.a. $\frac{dy}{dx}$).

a) $y = e^{\sin x}$

b) $y = \sinh(2x)$

c) $y = \ln(x^2 + x)$

d) $y = \int_1^x \frac{1}{\sqrt{t^2+t}} dt$

e) $y = \int_x^{2x} \frac{1}{t} dt$

f) By the time I turn 30, one thing I want to have done is . . .

2. Integrate the following:

a) $\int \frac{u}{u-1} du$

b) $\int \frac{e^{2x} - e^{-2x}}{2} dx$

c) $\int \frac{x^2 + 2}{x^3 + 6x} dx$

d) $\int \pi x^4 + e x^5 dx$

3. (You knew this was coming, didn't you?) State the Fundamental Theorem of Calculus (both parts). Briefly explain what it means.

4. Let $f(x) = e^{x^3} + 1$.

Find the inverse function, $g(x) = f^{-1}(x)$.

What is the domain of g ?

Find $f(0)$

Find $g'(1)$ in two different ways.

Method I

Method II

5. Let $h(x) = \operatorname{arcsec}(x)$. (If you don't remember — and can't figure out — the derivative of $\sec x$, you may purchase that information for 3 points.)

Find $h'(x)$

Find y' if $y = (\arcsin(\cos x))^3$

6. Here's a sketch of the curve given by $x = y^3 - y$. Find the area of the region above the x -axis that is bounded by this curve and the line $y = 3x$. Show your work.

7. Let A be the bounded region between the graphs of f and g as shown.

a) Write an integral that calculates the area of A .

b) Write an integral that gives the volume of the solid generated by spinning this region around the x -axis.

c) Explain why the integral in part b works. Be sure to explain each part of the formula.

Extra Credit: There's a move in Washington to change aspects of the health care/health insurance system in the US. Right now there are five bills in Congress that are being considered. For half a point each, name up to six major changes that are contained in these bills.