Name: ______________________

Final Opportunity — Fall '16

No calculators or cell phones are allowed — please turn them off and put them away. During the exam, you may not post anything anywhere (including YikYak) or get outside help in anyway (including that book someone left in the bathroom). If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in this exam. Remember to **check your work** whenever possible.

1. The foundational idea underlying all of Calculus is the limit. In this class we described the limit as "The end result of an infinite process." The limit was used to define both main concepts in this course: integrals and derivatives.

   Here's how the limit is used to define the integral with right endpoints and equal sized boxes:

$$\int_a^b g(x)\,dx = \lim_{N\to\infty} \sum_{i=1}^{N} g\left(a + \frac{(b-a)i}{N}\right)\frac{b-a}{N}.$$

   What's the "end result" in this case — that is, what is the limit being used to calculate? What's the "infinite process"? Draw a picture that helps you make sense of this formula.

   Here's one way to use the limit to define the derivative:

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}.$$

   What's the "end result" in this case — that is, what is the limit being used to calculate? What's the "infinite process"? Draw a picture that helps you make sense of this definition.

2. The Fundamental Theorem of Calculus states (roughly) that integrals and derivatives undo each other. Each statement below is an incorrect version of the FTC. Explain why it's wrong and correct the mistake.

$$\int_a^b \frac{df}{dx}\,dx = f(b)$$

$$\frac{d}{dx}\int_a^x g(t)\,dt = g(t)$$

3. Suppose that $f(x) = \displaystyle\int_0^x t\sin(t)\,dt$.

   Write the **definition** of $f'(x)$ for this function f; be sure to include a limit.

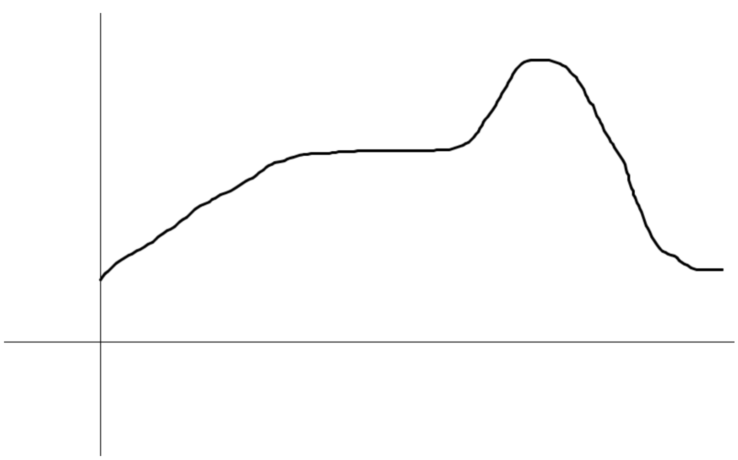
   Use the Fundamental Theorem of Calculus to find $f'(x)$.

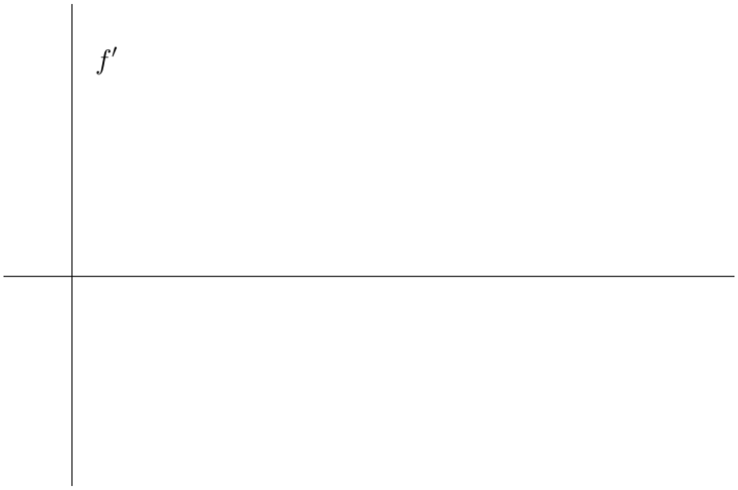
   Find $\displaystyle\frac{d}{dx}\int_0^{x^2} t\sin(t)\,dt$.

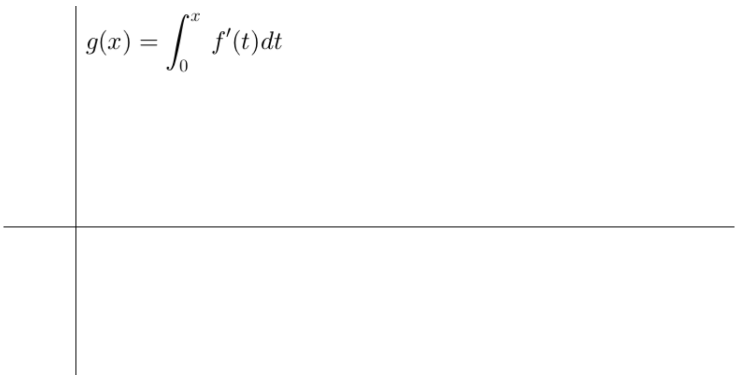
4. Ten years from now, what will you remember about this class?

5. The top axes shows the graph of $f(t)$. On the middle axes, sketch a graph of the derivative of f (that is, $f'(t)$ or $\frac{df}{dt}$).

   On the bottom axes, graph the accumulation function **for** $f'(x)$, $g(x) = \int_0^x f'(t)\,dt$. (Be sure corresponding points are lined up vertically.)



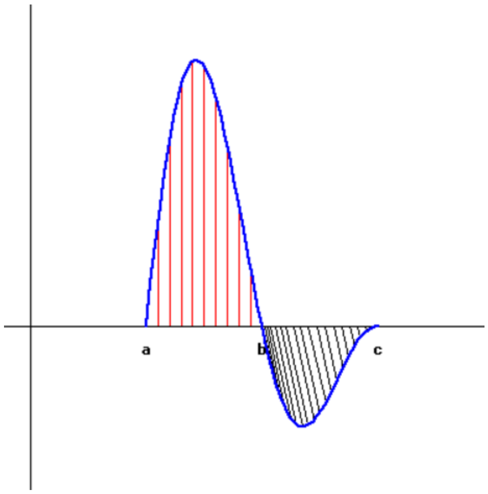




How are the top and bottom graphs related?

6. Find y' if
$$9(x^2 + y^2) = 3(x^3 - y^3).$$

7. In the picture here, the area under f with vertical shading (on the left) is 8; the area with diagonal shading (on the right) is 3.



Find the following:

(a) $\int_a^b f(x)\,dx =$

(b) $\int_c^b f(x)\,dx =$

(c) $\int_a^c 3f(x)\,dx.$

(d) $\int_a^c |f(x)|\,dx.$

(e) $\int_a^b f(x) + 3\,dx.$

8. The answer to each of the following is a number. Find it, showing your work. You do not need to simplify.

   (a) The area under one loop of a sine curve.

   (b) $\lim_{n\to\infty} \sum_{i=1}^{N} \frac{3}{N}[(\frac{3i}{N})^2 + 3(\frac{3i}{N})] =$

(c) The average rate of change of the function $f(t) = t^2 - 2t$ over the interval from $t = 1$ to $t = 3$.

(d) The instantaneous rate of change of the function $f(t) = 2t^2 - 2t$ at time $t = 1$.

(e) The x-value where $g(x) = \frac{x^3}{3} + 2x^2 - 5x + \pi$ has a point of inflection.

Extra Credit: Here are two famous — and frequently misinterpreted lines. Complete each one (1 point each), naming the person who penned it (1 point each).

"Two roads diverged in a wood, and ..."

"Down in the shadow of the penitentiary / Out by the gas fires of the refinery / I'm ten years burning down the road / Nowhere to run, ain't got nowhere to go / I was ... "