Name: _____

Opportunity III — Fall '16

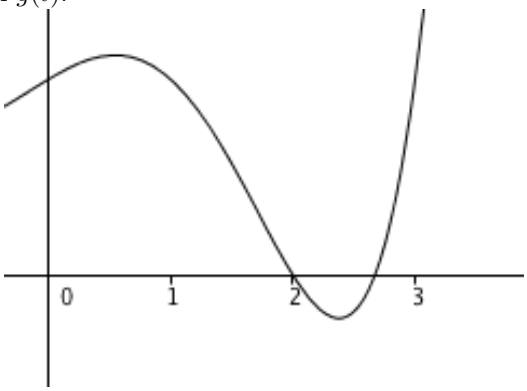
No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Savannah. If need be, she will text Dave and get a response. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. Find these sums:

$$\sum_{j=1}^{5}(5j+3)$$

$$\sum_{k=2}^{6}(k^2-4)$$

2. Here's the graph of a function $g(t)$:



To the right of each integral, give a brief description of what that integral calculates.

A $= \int_0^1 g(t)\,dt$

B $= \int_0^2 g(t)\,dt$

C $= \int_2^{2.5} g(t)\,dt$

D $= \int_0^2 2g(t)\,dt$

E $= \int_2^3 g(t)\,dt$

Now put them in order (using their letters) from least to greatest:

Order: _____ < _____ < _____ < _____

3. Let $k(x) = x^2 + 11 + \frac{6}{x}$. Find the minimum value of k for values of x greater than zero. Show your work and justify your reasoning.

4. Here's the formula for the Riemann Sum — which correctly calculates the area under the function f between $x = a$ and $x = b$:

$$\lim_{N\to\infty} \sum_{i=1}^{N} f(x_i^*)\Delta x_i.$$

Briefly describe what each part of this formula means — what it contributes to figuring out the area under the function f. Draw a picture on the right that illustrates as many of these parts as possible.

(a) N

(b) $\lim_{N\to\infty}$

(c) Δx_i

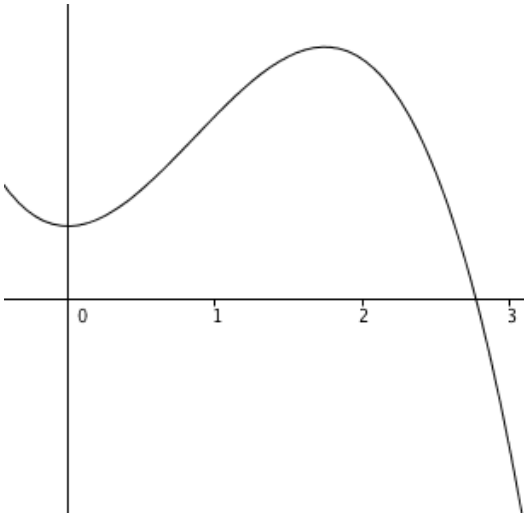
(d) x_i^*

(e) $f(x_i^*)$

5. Tell me about the most ambitious goal you have for your life.

6. Suppose you are approximating the area under the function g between $x = 0$ and $x = 3$ using 6 boxes and right endpoints. Write out your approximation in terms of g (for instance, $g(2)$ represents the height of the function g at $x = 2$). (You can write out the terms for most of the credit; for full credit also write it out with summation notation.)

Here's a graph of one possible function g. Draw boxes that would represent your work above — a Riemann Sum approximation to $\int_0^3 g(x)\,dx$ using 6 boxes and right endpoints.



7. Find **all** of the possible anti-derivatives for these two:

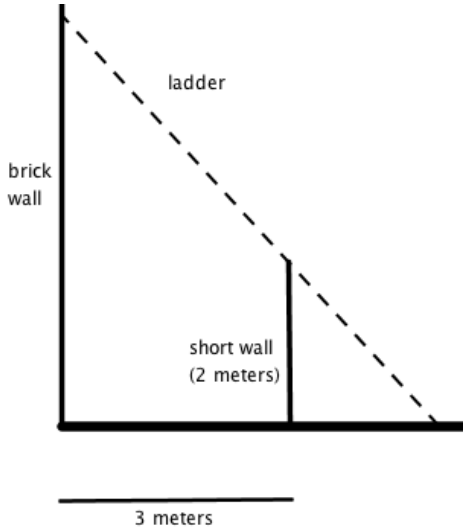
(a) $\int x^6 - \cos x\,dx =$

(b) $\int \sqrt{x}\,dx =$

For the function $g(x) = (x+1)^2$, find an anti-derivative $G(x)$ that satisfies $G(1) = 3$.

8. Set up (but **do not solve**) this optimization problem. You are done when you have a function of one variable along with the domain of interest.

You are trying to use a ladder up against a brick wall. Standing 3 meters away from the brick wall is a short wall (2 meters tall). What is the shortest ladder that will reach over the short wall and rest against the brick wall? Be sure to clearly define any variables you use and show your work.



9. You take a trip along a long, straight road, starting at home. From the time you leave in your 1986 Honda Civic (that's seen its better days), here's what happens:

- You drive for half an hour at 40 mph (the top speed for your old clunker).
- You realize you've forgotten your wallet, so you turn around, driving 40 mph for the next 15 minutes.
- Your car begins to sputter, and you gradually slow down over the next 10 minutes until you're only doing 20 mph.
- You limp home the rest of the way at 20 mph.

On the top graph, draw an accurate **position versus time** graph. On the lower graph, draw an accurate **velocity versus time** graph. Label both graphs — including units. **Be sure that the scales match up on both graphs.** Be as accurate a possible.

Extra Credit: If another country were flying drones over the US, occasionally using them to drop bombs, we would call that an act of war. By the same logic, the Obama administration has carried out wars in many countries, not just Iraq and Afghanistan. Name the five other countries the US has bombed with drones since 2009 (for half a point each).