Name: _____

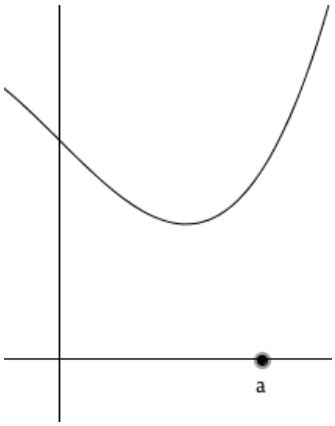
Opportunity II — Spring '16

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Savannah. If need be, she will text Dave and get a response. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. Here's a definition of the slope of f at the point a:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

Add to the picture below, illustrating all of the important parts of the formula (i.e. $x, f(x), f(a), x - a, f(x) - f(a)$, and $\frac{f(x)-f(a)}{x-a}$).



Why does the formula above include the $\lim_{x \to a}$? What purpose does that serve?

2. Here's a few warm-up problems. Find the derivative of each function.

a) $f(x) = 3x^2 - 4x^5$

b) $g(x) = x^2 \cos x$

c) $h(x) = \ln(x^2) + 3^{2x}$

3. Fill in the blank with a statement about a function on the interval $(0, 1)$. (Appropriate answers might include things like "increasing", "negative, " "concave down," or "can't tell".)

If f is positive, f' is _____.

If f is increasing, then f' is _____.

If f' is increasing, then f is _____.

If f' is negative, then f is _____.

If f' is negative, then f'' is _____.

If f' is increasing, then f'' is _____.

4. In each case, find y' in terms of the functions f, g, f', and g' (and, of course, x).

a) $y = f(x^2)$

b) $y = (f(x))^3$

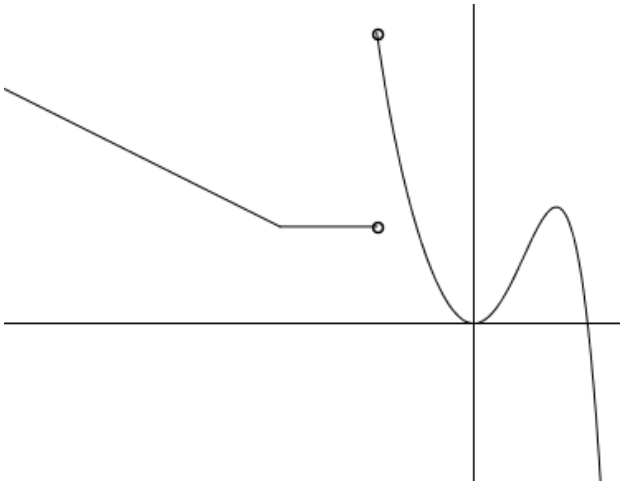
c) $y = f(g(x))$

d) $y = g(g(x) + f(x))$

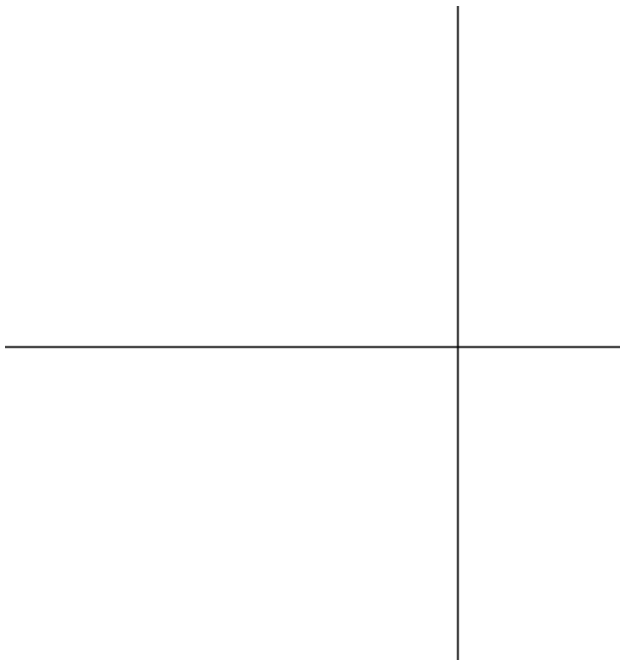
e) $y = \frac{f(x^3)}{g(x^3)}$

5. If you could change one thing about SMCM, what would it be?

6. The graph of a function $f(x)$ is shown on top. Sketch the derivative $f'(x)$ on the bottom.



$f(x)$

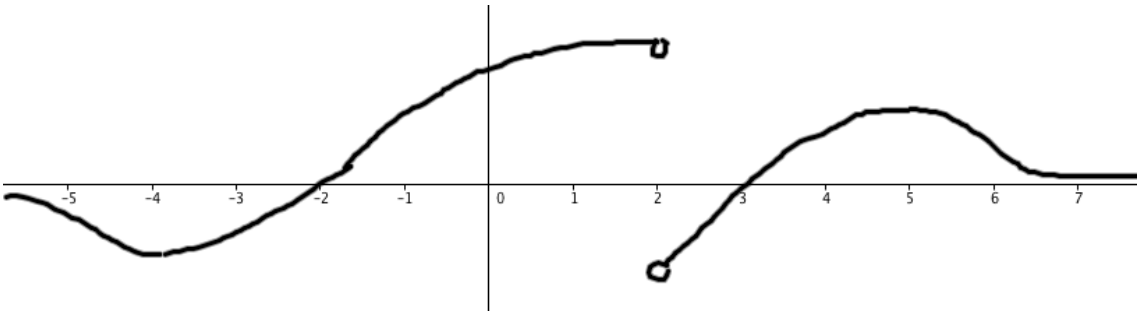


$f'(x)$

7. Suppose $g(x) = \arctan x$. Prove that $g'(x) = \dfrac{1}{1+x^2}$. (For partial credit, prove that $g'(x) = \frac{1}{\sec^2 g(x)}$.)

If $b(x) = \arctan(x^2)$, find $b'(x)$.

8. Here's a graph of g'(x), the **derivative** of $g(x)$, which is a **continuous** function.



For each of the attributes below, give **all** points where the property is true. In some cases this will require a single point or handful of points (separated by commas). In others, it will require an interval or collection of intervals.

a) g is increasing:

b) g is concave up:

c) Critical points of g:

d) Local maxima of g:

e) Local minima of g:

f) Critical points of g':

9. Find the absolute maximum and minimum (if they exist) of the function

$$g(x) = \frac{3}{2 + x^2}$$

on the interval $[-1, 2]$. Justify your answers.

10. To the left of each statement below, write the complete word "True" or the complete word "False."

a) If f is continuous at $x = 3$, then f is differentiable at $x = 3$.

b) If $g'(5) = 0$ and $g''(5) = -2$, then g has a local minimum at $x = 5$.

c) If $j'(2) = 0$, then j has a critical point at $x = 2$.

d) If k has a local maximum at $x = 7$, then $k'(7) = 0$.

Extra Credit: One of the nine members of the U.S. Supreme Court recently passed away. President Obama has nominated a replacement, but the leaders of the U.S. Senate are refusing to even consider the replacement. For one point each, name the recently deceased judge, the nominee, and the leader of the U.S. Senate.