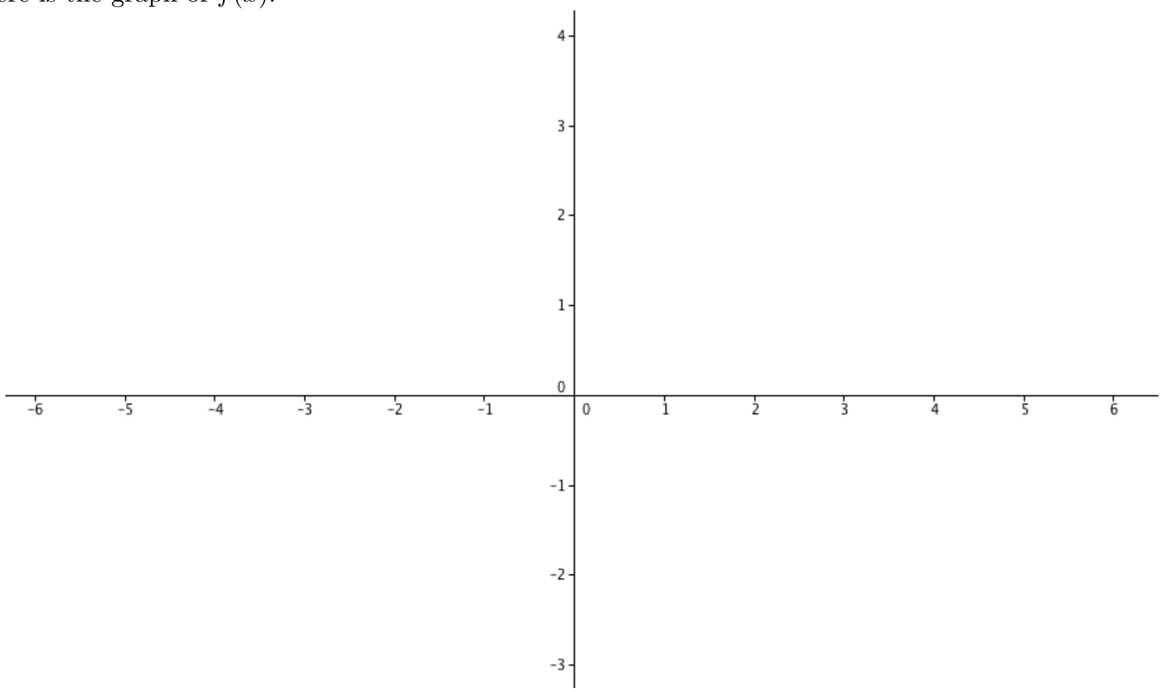
Name: ____________________

Opportunity I Fall '16

No calculators or cell phones are allowed — please turn them off and zip them away. If you have any questions, please ask Savannah. If she can't answer, she'll get a response from Dave shortly. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. Here is the graph of $f(x)$:



Find the following (or state DNE):	Name a point (if one exists) where ...
$\lim_{x\to 3} f(x) =$	... f is defined but not continuous:
$\lim_{x\to 0^+} f(x) =$	... f is continuous but not differentiable:
$\lim_{x\to -2^+} f(x) =$	... $f > 0$, $f' > 0$, and $f'' < 0$:
$\lim_{x\to -3} f(x) =$	... f is differentiable but not continuous:
	f' is increasing:

2. Find the following limits:

$$\lim_{x\to 1} \frac{x^2 - 3x + 2}{x - 1} =$$

$$\lim_{y\to 4} \frac{y^2 - 3y - 4}{\sqrt{y} - 2} =$$

3. Suppose $C(t)$ represents the concentration of carbon dioxide (CO_2) in earth's atmosphere t years after the year 1800, and is measured in parts per million (ppm). (We are ignoring the annual fluctuations of CO_2 — so we may assume C is a smooth function.) Here's what we know about C:

- in 1800, the atmosphere was at 280 ppm
- the concentration of CO_2 rose slowly (but steadily) until 1870
- it rose a bit faster from 1870 until about 1940
- it dipped slightly during World War II, and started rising again afterwards
- after 1950, it grew faster and faster
- the concentration recently passed the 400 mark.

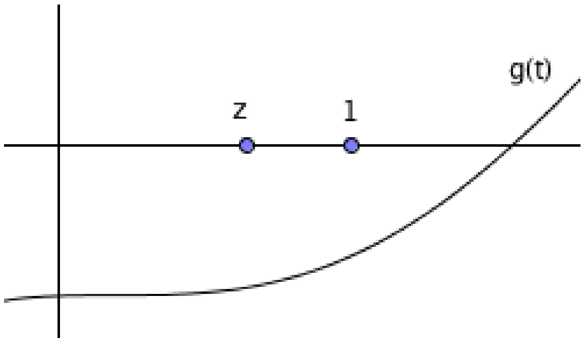
On the axes below, sketch an accurate graph of $C(t)$ that contains all the information above, adding labels and points as needed.

Suppose we know that $C(190) = 352$. Write a sentence (including units) that explains what this equation means.

Suppose we also know that $C'(190) = 0.6$. Write a sentence (including units) that explains what this equation means.

Use the facts that $C(190) = 352$ and $C'(190) = 0.6$ to estimate the CO_2 concentration in 1992.

4. Here is the graph of the function $g(t)$.

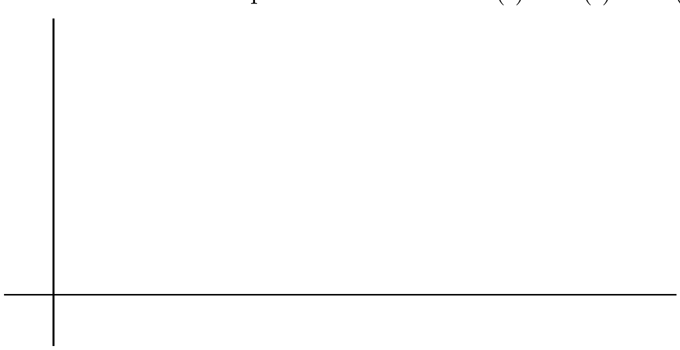


The point z is always to the left of 1. This question asks what happens when z is moves to the right, closer to 1 (without ever reaching it). Complete the chart, with "+" or "-" in the left column (indicating if the quantity is positive or negative) and "Inc." "Dec." or "Const." (indicating if the quantity is increasing, decreasing, or constant as z moves to the right. The first row is filled in.

Quantity	$+\ /\ -$	Inc. / Dec. / Const.
z	$+$	Inc.
$g(z)$		
$g(1)$		
$g(1)-g(z)$		
$1-z$		
$\frac{g(1)-g(z)}{1-x}$		
$\lim_{z\to 1^-} \frac{g(1)-g(z)}{1-z}$		

5. What has been the happiest moment of your college career so far?

6. The graph of a position function $s(t)$ is shown on the top graph. Sketch a graph of the velocity $v(t) = s'(t)$ on the middle axes. Graph the acceleration $a(t) = v'(t) = s''(t)$ on the bottom graph.



7. Assume that f is a smooth function defined between 0 and 5, taking on the values below. Assume f does not misbehave (or do anything crazy) between the points listed.

x	0	1	2	3	4	5
$f(x)$	1	9	16	21	25	27

For each statement below (and considering only $[0, 5]$) , write "True", "False". Briefly explain your answer.

a) f is positive.

b) f' is negative.

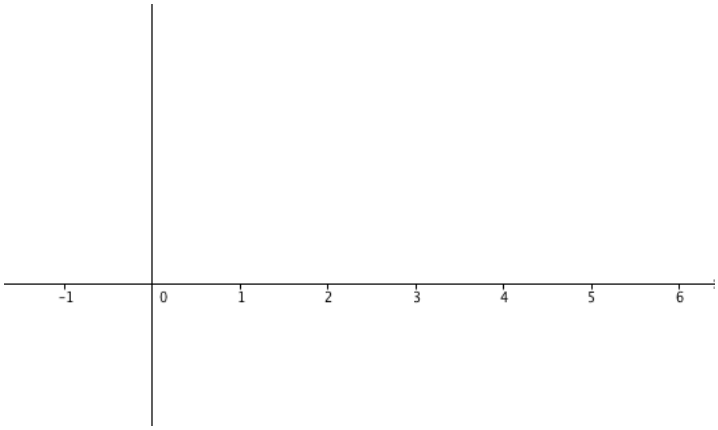
c) f'' is negative.

d) f is increasing.

e) f' is increasing.

8. If $k(x) = x^2 - 3x$, show (using a definition of the derivative) that $k'(x) = 2x - 3$.

9. Here is the graph of $j'(x)$ (that is, the **derivative** of the function $j(x)$.)



For each statement below, give an x-value where that statement holds, along with a brief explanation of your thinking.

j is greatest at ...

j' is greatest at ...

j'' is greatest at ...

j is smallest at ...

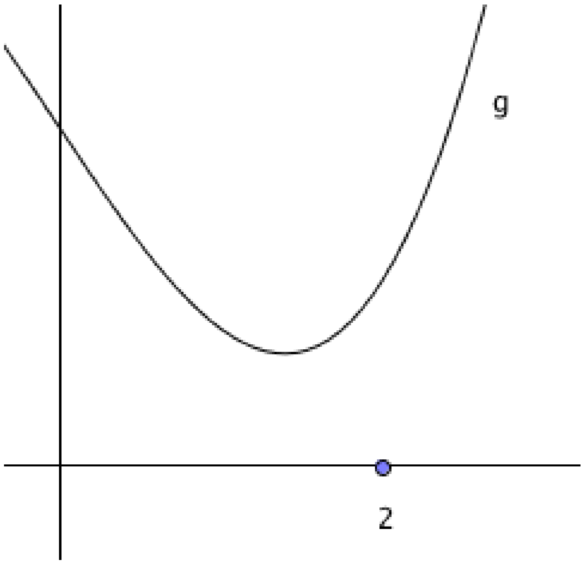
j' is decreasing at ...

j is concave up at ...

10. Here is a perfectly good definition of the slope of $g(x)$ at the point $x = 2$:

$$\lim_{h \to 0} \frac{g(2) - g(2-h)}{h}.$$

Complete this picture, showing all relevant parts of the above formula.



What is the difference between $\frac{g(2)-g(2-h)}{h}$ and $\lim_{h\to 0} \frac{g(2)-g(2-h)}{h}$? Explain what each means.

In class we described a limit as "the end result of an infinite process." In the case of the definition above, what is the infinite process? What is the end result?

Extra Credit: Recently researchers in astro-physics announced that someone's prediction has been confirmed. For a point each, name the phenomenon they confirmed, the original person whose equations predicted it, and what (crazy) event they were observing.