Name: _____

Final Opportunity — Spring '13

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. You may not get help from anyone else, in any way. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam. Remember to **check your work** whenever possible.

Useful formulas:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(fg)' = f'g + fg' \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(f(g(x)))' = f'(g(x))g'(x)$$

$$(x^n)' = nx^{n-1}$$

1. (a) State the mathematical definition of the *derivative* of a function $f(x)$ at $x = 7$.

   (b) Use a picture to show why the definition of the derivative gives the slope of f at $x = 7$.

   (c) Write down the **definition** of the derivative of $g(x) = 3x^2 - 2x$ at the point $x = 1$.

   (d) Show (using the definition of the derivative, and not the power rule or any other derivative rules) that $g'(1) = 4$.

2. Compute the derivatives of the functions below, using derivative rules. Do not simplify.

(a) $f(x) = x^5 \sin x$

(b) $g(x) = \sqrt{x^2 + 3x}$

(c) $h(x) = \dfrac{1 + \tan x}{\cos x}$

3. The middle axes shows the graphs $y = f'(x)$.

   On the top axes, sketch a graph of a possible function $y = f(x)$.

   On the bottom axes, sketch a graph of $y = f''(x)$.

4. Write $\int_0^2 (3x^2 + 4x)\, dx$ as a limit, using right endpoints in the Riemann sum.

Using only that definition (and not the Fundamental Theorem of Calculus), calculate the area under the function $y = 3x^2 + 4x$ between $x = 0$ and $x = 2$.

Check your answer by using the Fundamental Theorem or Calculus.

5. (30 points) Below is the graph of the function f. Consider the function that accumulates the area under f,

$$g(x) = \int_0^x f(t)\, dt$$

Answer these questions about g and f:

(a) $g(2) =$

(b) $g'(2) =$

(c) g has a local max when $x =$

(d) True or False: $g(3) > g(2)$? (Circle one: T or F)

(e) At the point $x = 2$, g has a ...

(f) Give the largest set on which g is concave up:

(g) Find $\frac{d}{dx} \int_0^{x^2} f(t)\, dt$.
Hint: Chain Rule

6. Answer the following questions about integrals.

(a) $\displaystyle\int_1^4 2x + 3\sqrt{x}\, dx =$

(b) $\displaystyle\int_0^{2\pi} \sin x\, dx =$

(c) The following is a definite integral in disguise. What integral is it? (You do **not** have to do the integral.)
$\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \tan\left(\frac{3i}{n}\right)\frac{3}{n} =$

(d) Is $\displaystyle\int x\cos x\, dx = x\sin x + \cos x + C$? Why or why not?

(e) $\displaystyle\int x^2\sqrt{5+2x^3}\, dx =$

7. Optimization

   One of the applications of Calculus we worked on this semester was optimization. Suppose you have something you want to optimize, let's call it Blob. You want to figure out how to make Blob as big as possible.

   What are the steps you would go through to use Calculus to do this?
   (Bonus points if you reference *Casablanca*.)

8. Write the Fundamental Theorem of Calculus:

   (a) Part I:

   (b) Part II:

   (c) If you were explaining to a friend who had never studied calculus exactly what the FTC means, what would you say? Write 3-4 sentences that would help a non-Calculus student understand the importance of the FTC.

Extra Credit: For $\frac{1}{4}$ point each, name as many countries as you can that have just four letters (when written in English). (You get a bonus point if you name all 10.)