

Name: _____

OPPORTUNI3

SPRING '13

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Dave or Demara. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. The graph of $f'(x)$ is shown on the middle axes. Graph $f(x)$ on the top axes and $f''(x)$ on the bottom axes.

2. The Mean Value Theorem states something about a function f that is continuous and differentiable on an interval $[a, b]$. Draw a picture to illustrate the MVT.

Complete this sentence, using the same notation used in the picture above: “The Mean Value Theorem says that there exists a point $c \in (a, b)$ with the property that...”

3. Fill in the blank with a statement about a function on the interval $(0, 1)$. Write a sentence explaining your reasoning. (The correct answer might "undetermined" meaning that the given information doesn't determine anything about the function in question.)

If f' is positive, f is _____.

If f' is increasing, then f is _____.

If f' is decreasing, then f'' is _____.

4. Here is the graph of a function $f'(x)$.

Answer the following questions about related functions:

a) What are the critical points of f ? Why?

b) Where does f have local maxima? Why?

c) Where does f have inflection points (i.e. where f changes from concave up to concave down or from concave down to concave up.) Why?

5. You have a problem. You are a farmer with two bulls that don't get along. On your property, you already have two brick walls that meet, and you decide to put up fencing (with each section parallel to one of the walls) to create two pens. You have 200 m of fencing and decide that your pens will look something like this:

What fence lengths will maximize the area for your two bulls? Explain your reasoning.

6. For the function $f(x) = x^3 - 6x^2 + 9x + 2$, find the x -values that give the local and absolute maxima and minima on the interval $[-1, 5]$. Show your work (including any graphs you use).

Local maxima:

Local minima:

Absolute maxima:

Absolute minima:

7. Find all anti-derivatives for the following functions:

a) $f(x) = x^3 + \sin x$

b) $g(x) = \sqrt{x}$

c) $h(x) = \frac{\pi}{x^3}$

d) $j(x) = \cos(3x)$

e) $h(x) = (1 - x)^4$

f) Three things I want to have done before 2023 are...

Extra Credit: If you rank cities (including their surrounding metropolitan areas) by population, the top of the list is Tokyo with around 32 million people. For a half-point each, name the next four.