Name: _____

Oppor2nity

Spring '13

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Dave or Demara. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. Use the two sets of axes to draw a graph that describes my commute to campus today (more on that below). On the top axes, graph the distance from home (along a straight road) as I bike to school, which is 1.5 miles away from home. On the bottom axes, graph my velocity. On both, the horizontal axis measures time, starting at 8am, measured in minutes.

Here's this morning's story:

Ellie woke us all up early this morning, so I had plenty of time to get to campus. On my way out the door at 8am, I grabbed an apple for breakfast and hopped on my bike, riding slowly as I ate. Half a mile later, I realized that my lunch was sitting on the counter, so I went back to get it. Halfway back to the house, I finished the apple and sped up a bit. At home, I grabbed my lunch, took a minute to kiss Ellie goodbye (again), and headed out, now worried that I'd be late for my 8:15 meeting. Halfway to school, I had to slam on my brakes to avoid a stray cat. After that I really had to bike as fast as I could; I arrived at school just in time for my meeting.

Be as accurate as possible, given the information you have.

2. Using the derivatives of $\sin(x)$ and $\cos(x)$, prove that $(\tan x)' = \sec^2 x$.

3. Suppose that f and g are positive functions and that both f' and g' are always negative. Let $h(x) = f(g(x))$. Is h also a positive function? Explain your reasoning.

Is h' always negative? Always positive? Sometimes positive and sometimes negative? Explain your reasoning.

4. Here is the formula for the derivative of a function f at a point a:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Draw a picture to illustrate why this formula correctly determines the slope of f at $x = a$. Explain your thinking. Be sure to explicitly explain each part of the formula.

According to the definition of the limit above, x might approach a from the left side, and it might approach from the right side. Explain why the formula still works in both cases (even though your picture only depicts one of them).

5. Using the definition of the derivative that has $h \to 0$, write out a limit that would give the slope of the function $f(x) = x^3 \sec x$ at the point $x = 1$. Please, please do not try to simplify this limit.

If you tried to evaluate this limit by plugging in $h = 0$, it would have the form $\frac{0}{0}$. That shouldn't surprise you — why not?

As practice for the Gateway test, find $f'(x)$ using the various derivative rules.

6. [This problem isn't too far from a real problem scientists did after the 2010 Deep Water Horizon oil spill in the Gulf of Mexico.]

Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the area of the spill is increasing at a rate of $100\frac{m^2}{sec}$, how quickly is the radius increasing when the spill has a radius of $200m$?

Extra Credit: If some other country were flying drones over the US, taking pictures or shooting people they claimed were fighters, we would call that war. By this definition, the US is engaged in war in many more countries than just Afghanistan & Iraq. For a half point each, name all six.