Name: _____

Opportunity I

Spring '13

So exciting — your first opportunity to show what you've learned in Calc I!

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Dave or Demara. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. In class we described a limit as "the end result of an infinite process."

Describe in your own words what this means.

Explain what a limit has to do with this equation:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \ldots = 1.$$

2. The graph of the function $f(x)$ is shown here.

On the axes below, graph the following functions. For each, be sure to indicate several points on each graph.

a) $h(x) = f(x - 3)$

b) $g(x) = f(x + 1) + 2$

c) $k(x) = 2f(x) - 1$

3. The graph of the function $g(x)$ is picture below.

$\lim_{x\to -3} g(x) =$ $\lim_{x\to 2^+} g(x) =$ $g(4) =$

$\lim_{x\to 2^-} g(x) =$ $g(-2) =$ $\lim_{x\to 4} g(x) =$

$\lim_{x\to 2+} g(x+2) =$ $\lim_{x\to -2^-} g(x^2) =$ $\lim_{x\to 2} x\, g(x) =$

List all the points where g is discontinuous:

4. Let

$$h(x) = \begin{cases} \frac{2x^2-8}{x-2} & \text{if x } \neq 2 \\ ? & \text{if x } = 2. \end{cases}$$

What should you choose for the value of $h(2)$ so that h is a continuous function for all x? Why?

5. In the picture here, the point P is fixed and never moves. The point Q moves along the graph of the function. For each of the following, describe in a word or several words what happens as Q moves toward P along the curve — getting arbitrarily close but never reaching P.

Q_x, the x-coordinate of Q:

Q_y, the y-coordinate of Q:

$\Delta_x = P_x - Q_x$:

$\Delta_y = P_y - Q_y$:

$\frac{\Delta_y}{\Delta_x}$:

6. What's the best class you've taken at SMCM (other than this one) — and why was it so great?

7. Consider the following formula:

$$\lim_{t \to 5} \frac{f(5) - f(t)}{5 - t}.$$

Will this formula correctly calculate the slope of f at the point $t = 5$? Why or why not? If you think it does, explain why. If not, make an adjustment to the formula so that it is correct. (Do not simply state a correct formula which is very different from this one.) Add details to the above picture to help illustrate your answer.

8. Explain how to use the Limit Laws to calculate the following limit. What is that limit?

$$\lim_{x \to -2} \frac{3x^2 - \pi x + 42}{(x - 2)(3x + 1)}.$$

9. Using the definition, calculate the instantaneous change of the function $f(x) = -3x^2$ at $x = 2$. Show your work. (You will receive no credit for simply taking the derivative and plugging in $x = 2$.)

10. Let $g(t)$ represent the amount of oil we expect to remove from below the earth's surface in year t. The units of g are in millions of barrels per day, and the horizontal axis (t) is measured in years. Currently, we are removing about 89 millions of barrels per day — so $g(2013) = 89$ — but that number is growing. According to many experts, that number will peak in 2030 (at approximately 100), then slowly fall, and then fall more quickly until 2070.

Draw a sketch of what $g(t)$ might look like on the given axes.

If we calculate the slope of g, what unit is the slope in?

Suppose the slope is currently 2 (in whatever units you described above). Explain what this means.

Put the following numbers in order from least to greatest. (If you are unsure about two values, put them together like we did in class.)

a) the average change of g between 2013 and 2030
b) the instantaneous change of g at 2013
c) the average change of g between 2013 and 2035
d) the instantaneous change of g at 2030
e) the average change of g between 2030 and 2045
f) the instantaneous change of g at 2045

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11. Let

$$f(x) = \begin{cases} 4 - x^2 & \text{if x } \leq 2 \\ x - 1 & \text{if x } > 2. \end{cases}$$

For each of the following, find it, or state that it doesn't exist.

a) $\lim_{x \to 2^-} f(x)$

b) $\lim_{x \to 2^+} f(x)$

c) $f(2)$

d) $\lim_{x \to 2} f(x)$

Extra Credit: On this day in 1996, the world's best chess player lost the first game in a six-game match to a computer. For a point each, name the chess player, the computer, and the company that made (and programmed) the computer.