Name: ______________________________

FINAL OPPORTUNITY FALL '15

No calculators or cell phones are allowed — please turn them off and put them away. During the exam, you may not post anything anywhere (including YikYak) or get outside help in anyway (including that book someone left in the bathroom). If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in this exam. Remember to **check your work** whenever possible.

Useful formulas:

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2} \qquad \sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6} \qquad (fg)' = f'g + fg' \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

1. The foundational idea underlying all of Calculus is the limit. In this class we described the limit as "The end result of an infinite process." The limit was used to define both main concepts in this course: derivatives and integrals.

   Here's how the limit is used in the definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

   What's the "end result" in this case — that is, what is the limit being used to calculate? What's the "infinite process"? Give the definition and draw a picture that helps you make sense of this definition.

   Here's how the limit is used to define the integral:

$$\int_a^b g(x)\,dx = \lim_{n \to \infty} \sum_{j=1}^{n} g(x_j^*)\Delta x_j.$$

   What's the "end result" in this case — that is, what is the limit being used to calculate? What's the "infinite process"? Give the definition and draw a picture that helps you make sense of this formula.

2. Write the definition of $f'(x)$ when $f(x) = 2x^2 + 1$.

Use the definition to prove that $f'(x) = 4x$.

3. Use right-hand endpoints and four boxes to approximate $\int_0^2 4x\,dx$.

Write the definition of $\int_0^2 4x\,dx$ as a limit, using right endpoints in the Riemann sum.

Use the definition (not the FTC, nor basic geometry facts) to prove that this integral is equal to 8.

4. The Fundamental Theorem of Calculus states (roughly) that integrals and derivatives undo each other. Finish a more precise statement of the FTC by completing both of the formulas below.

   Part 1. $\displaystyle\int_a^b \frac{df}{dx}\, dx =$

   Part 2. $\displaystyle\frac{d}{dx}\int_a^x g(t)\, dt =$

5. Using the FTC, find the following:

   (a) $\displaystyle\frac{d}{dx}\int_1^x \sin(t^2)\, dt =$

   (b) $\displaystyle\int_1^3 x^3 + \sin x\, dx =$

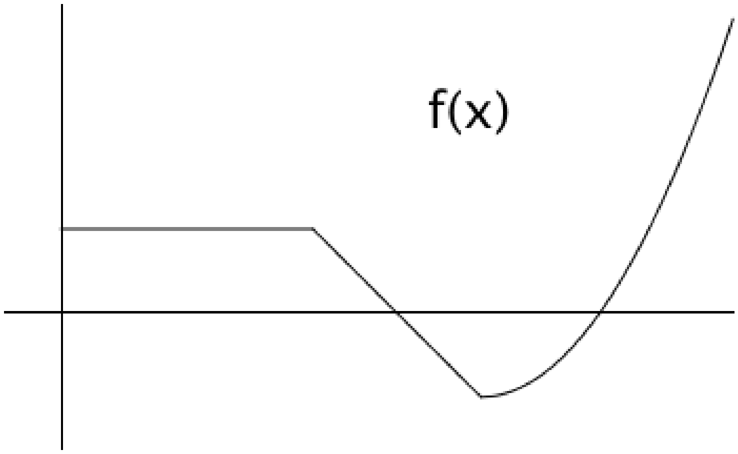
   (c) $\displaystyle\int_0^1 e^x + \pi\, dx =$

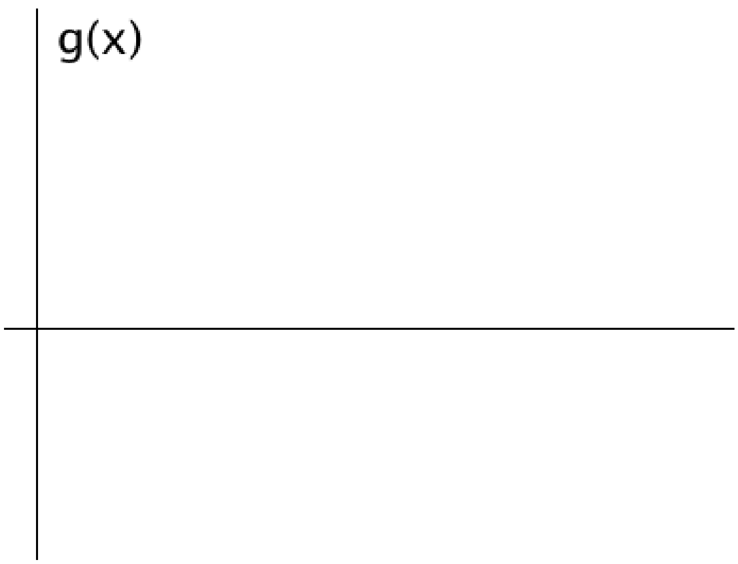
   (d) $\displaystyle\frac{d}{dx}\int_1^{\cos x} \frac{1}{1+t^2}\, dt =$

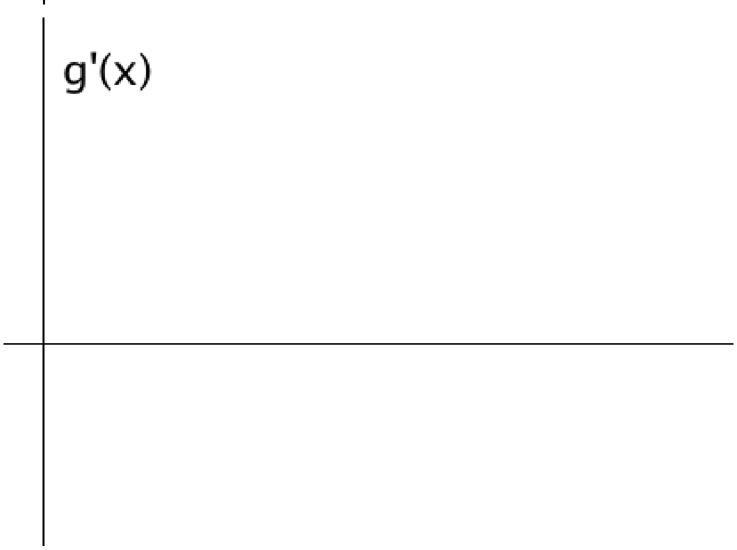
6. What's the most interesting thing you learned this semester in any class?

7. The top axes shows the graphs $y = f(t)$. On the middle axes, sketch a graph of the accumulation function $g(x) = \int_0^x f(t)\, dt$. (Be sure corresponding points are lined up vertically.)

   On the bottom axes, sketch a graph of $g'(x)$.







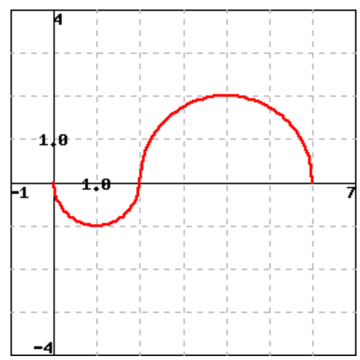
8. The answer to each of the following is a number. Find it. (You do not need to simplify.)

(a) The instantaneous rate of change of $f(x) = \frac{x}{1+x^3}$ at $x = 1$.

(b) The area under $g(x) = x^3 - x$ between $x = 0$ and $x = 2$.

(c) $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{5}{n} \sin(\frac{5j}{n}) =$

(d) The slope of $y = e^{3x}$ at $x = 0$.



(e) $\int_0^6 f(x)\,dx$, given f on the right, consisting of two semi-circles:

(f) The positive x-value that maximizes the function $f(x) = x \ln x$.

Extra Credit: Climate talks recently concluded with a landmark agreement to limit emissions of gases that cause climate change. For one point each, name the location of the talks, the country that emits the most such gases, the country that emits the most such gases per person, and the number of degrees (in Celsius) the agreement aims to cap warming at (in comparison to pre-industrialization temperatures).