Name: ______________________________

Opportunity III — Fall '15

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Sarah. If need be, she will text Dave and get a response. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. Here is the graph of the derivative of a continuous function, $f(x)$ (that is, the graph shows $f'(x)$):

Give a point where each of the following occurs, or state that no such point exists. Briefly state your reasoning.

a) f has a local maximum.

b) f' is decreasing.

c) f has a local minimum.

d) f has a point of inflection.

e) f is decreasing.

2. For the bottle on the left, sketch the graph of the height of water in the bottle as a function of the volume of water in the bottle.

3. Suppose g is a continuous function on the interval $[-3, 3]$, with all of the following properties:

- g has a root at $x = 0$, and that is its only root.
- g' is decreasing on the entire interval $[-3, 3]$.
- $g'(2) = 0$.

a) True or False: On the interval $[-3, 3]$, g reaches its maximum and minimum values. Explain.

b) If it reaches its maximum, where does that happen? Why?

c) If it reaches its minimum, where does that happen? Why?

4. Suppose $k(t) = t^4 + 2t^3$. Find the critical points of k.

On each resulting interval (to the left of the smallest critical point, in between any neighboring critical points, to the right of the largest critical point), determine if k is increasing or decreasing (i.e. draw a sign chart for the derivative, k').

Find the points of inflection of k.

5. Suppose another oil rig in the Gulf of Mexico explodes, leaving oil gushing into the water. The oil spreads out in a circle with the area increasing at a rate of $5\frac{km^2}{hr}$.

How quickly is the radius of the spill increasing when the radius reaches 10 km? (Be sure to include units.)

One way to contain an oil spill is to put floats around the entire spill (i.e. around the circumference). How quickly is the circumference changing when the radius is 10 km? (Again: use units!)

6. Fill in the blank with a statement about a function on the interval $[0, 2]$.

If f is positive, f' is ____________________________.

If f is concave up, then f'' is ____________________________.

If $f'(1) = 0$ and $f''(1) > 0$, then f ____________________________.

If f' is decreasing, then f is ____________________________.

7. How fast a water wave moves depends on the wavelength of the wave. For waves in deep water, the velocity (v) depends on positive constants (c and k) as well as the wavelength (λ) like this:

$$v(\lambda) = k\sqrt{\frac{\lambda}{c} + \frac{c}{\lambda}}.$$

For what wavelength is the wave velocity minimized? Explain your reasoning.

8. Set up (but **do not solve**) this optimization problem.

A pumping station will be shared by two villages, with a pipeline built to take water directly to each village as shown. Where along the river should the station be built to make the length of the pipe as short as possible?

(Your answer will earn you full credit if the meaning of any variables is clear, a function to be optimized is clearly defined, and a domain is clearly stated. You need not do the optimization — that is, you need not take any derivatives.)

Extra Credit: A group in the news lately prefers not to be called "Daesh" (which is an acronym). Give the other two acronyms this group would prefer to be known by, with a bonus point if you know what either acronym stands for.