Name: ______________________________

Opportunity II

Fall '15

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Sarah. If need be, she will text Dave and get a response. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. A friend claims that $(\sin x)' = -\cos x$, but you are certain that $(\sin x)' = \cos x$. What evidence would you show her to convince her that you are correct?

2. Here's a few warm-up problems. Find the derivative of each function.

a) $f(x) = 4x^3 + \cos x$

b) $g(x) = \sqrt{x \tan x}$

c) $h(x) = e^{2x} + \ln(2x)$

3. Using the derivatives of $\sin(x)$ and $\cos(x)$, prove that $(\cot x)' = -(\csc x)^2$.

4. Fill in the blank with a statement about a function on the interval $(0, 1)$. (Appropriate answers might include things like "increasing", "negative, " "concave down," or "Can't Tell".)

If f' is positive, f is ____________________________.

If f is concave up, then f'' is ____________________________.

If f'' is increasing, then f' is ____________________________.

If f' is negative, then f is ____________________________.

If f' is negative, then f is ____________________________.

If f' is increasing, then f is ____________________________.

If f' is increasing, then f is ____________________________.

5. In each case, find y' in terms of the functions u, v, u', and v' (and, of course, x).

a) $y = x^2 u(x)$

b) $y = u(v(x))$

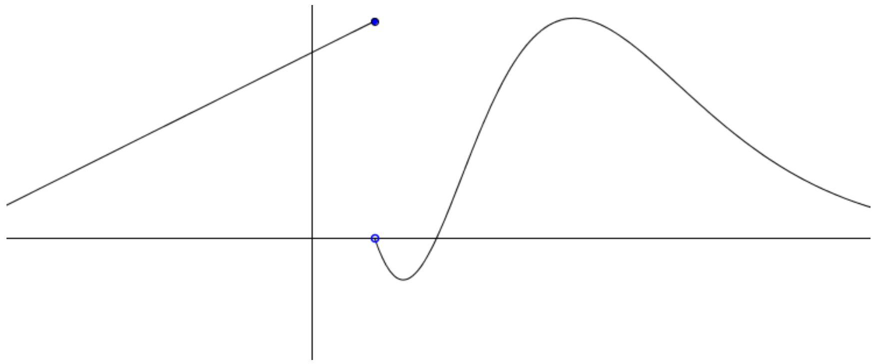
c) $y = u((u(x))^2)$

d) $y = u(x) + 3v(x)$

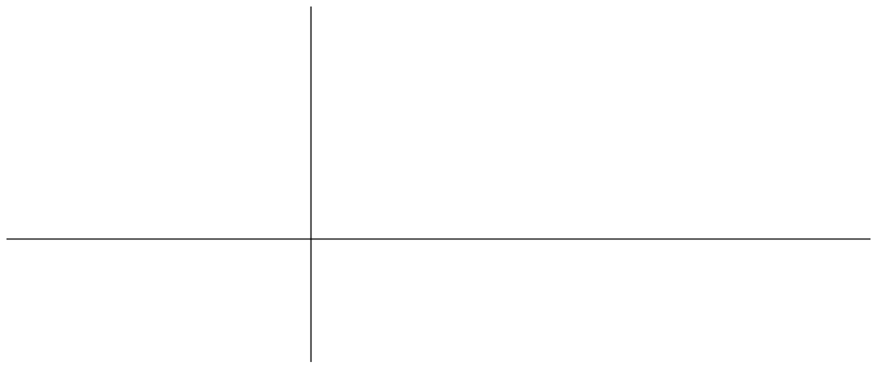
e) $y = \frac{v(x)}{\sin(u(x))}$

6. What's the second-most interesting class you're taking now (after Calculus, of course)? What makes it so interesting?

7. The graph of a function $f(x)$ is shown on the top graph. Sketch the derivative $f'(x)$ on the bottom axes.



$f(x)$



$f'(x)$

8. Suppose $g(x) = \arctan x$. Prove that $g'(x) = \dfrac{1}{1+x^2}$.

If $b(x) = x\arctan(k(x))$, find $b'(x)$ (in terms of k, k' and other functions involving x).

9. The goal of this problem is to prove the Power Rule ($(x^a)' = ax^{a-1}$) in the case when a is a fraction (using the fact that it works when a is an integer).

Start by setting $y = x^{\frac{p}{q}}$. (The goal is to show that $y' = \frac{p}{q}x^{(\frac{p}{q}-1)}$.)

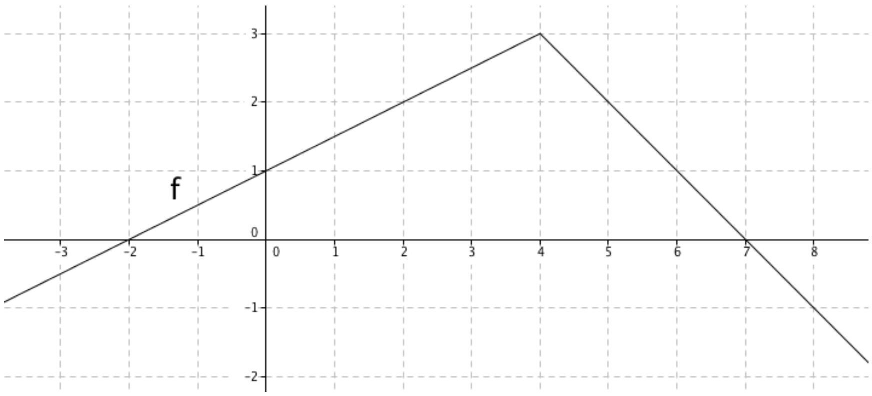
Raise both sides of this equation ($y = x^{\frac{p}{q}}$) to the power q:

Differentiate implicitly:

Solve for y':

Put the result in terms of x, simplifying to get the desired derivative:

10. Suppose the function $g(x) = x^2$, and the function $f(x)$ is given graphically here:



Let $h(x) = f(g(x))$ and $j(x) = g(f(x))$. Find the following, writing "DNE" if one doesn't exist.

a) $h(2) =$

b) $h'(2) =$

c) $j(6) =$

d) $j'(6) =$

Extra Credit: Wealth and income are increasingly concentrated in the hands of a few — who have money to spend on lavish things. In the last year, new record prices have been set for the private auction of a painting (\$179M), a sculpture (\$141M), and a violin (about \$10M). For one point each, name the artisan who made each of the items fetching a record price.