

## OPPORTUNITY I

FALL '15

Yay!! An Opportunity! So exciting!

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

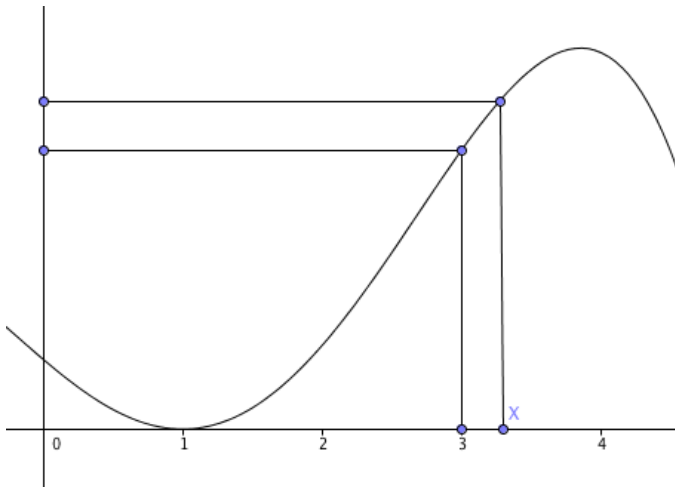
1. In class we described a limit as “the end result of an infinite process.”

Describe in your own words what this means.

Explain what this idea of the limit has to do with finding the derivative of a function at a point.

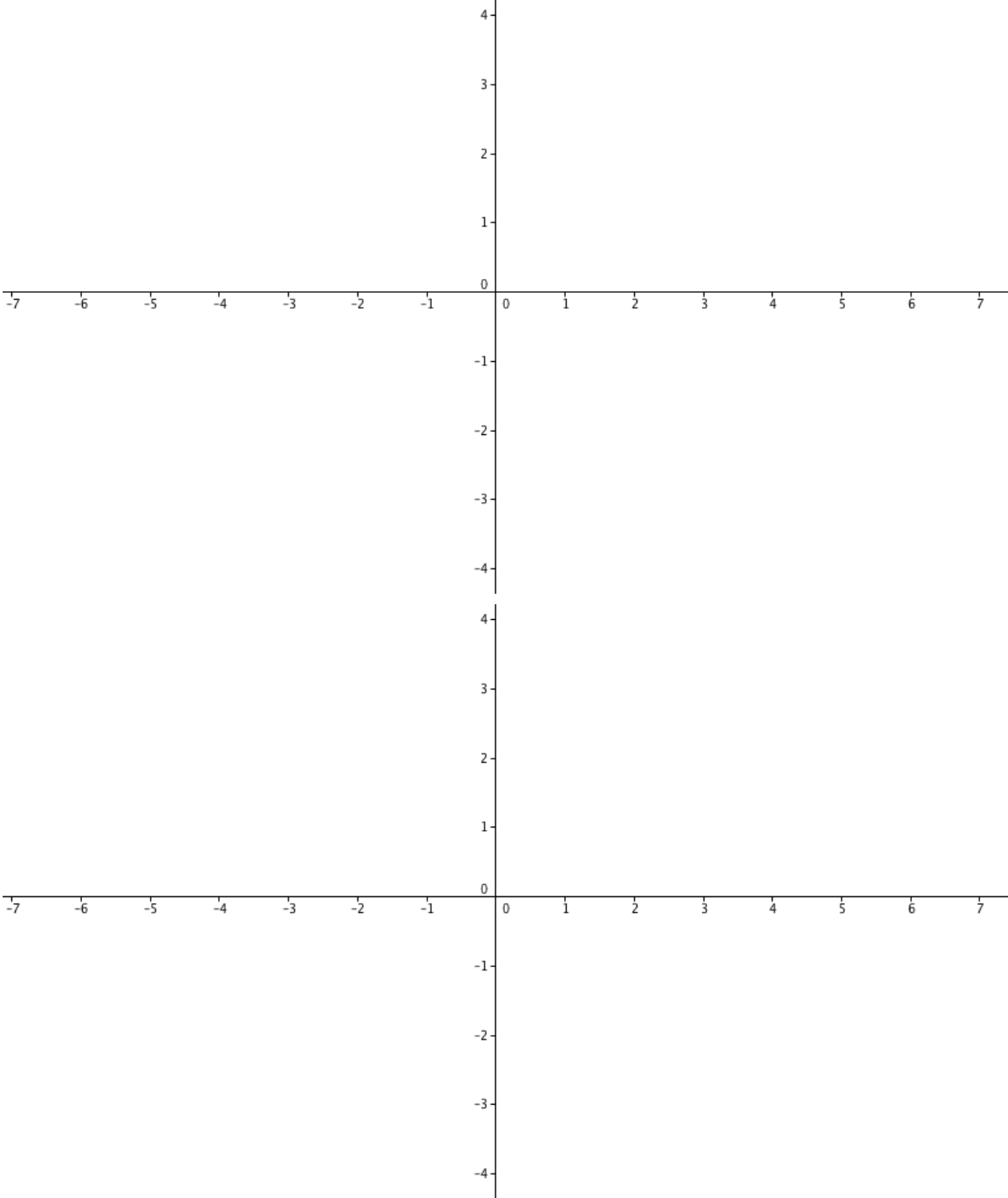
2. Consider the graph of  $f$  below and the formula

$$\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x}$$

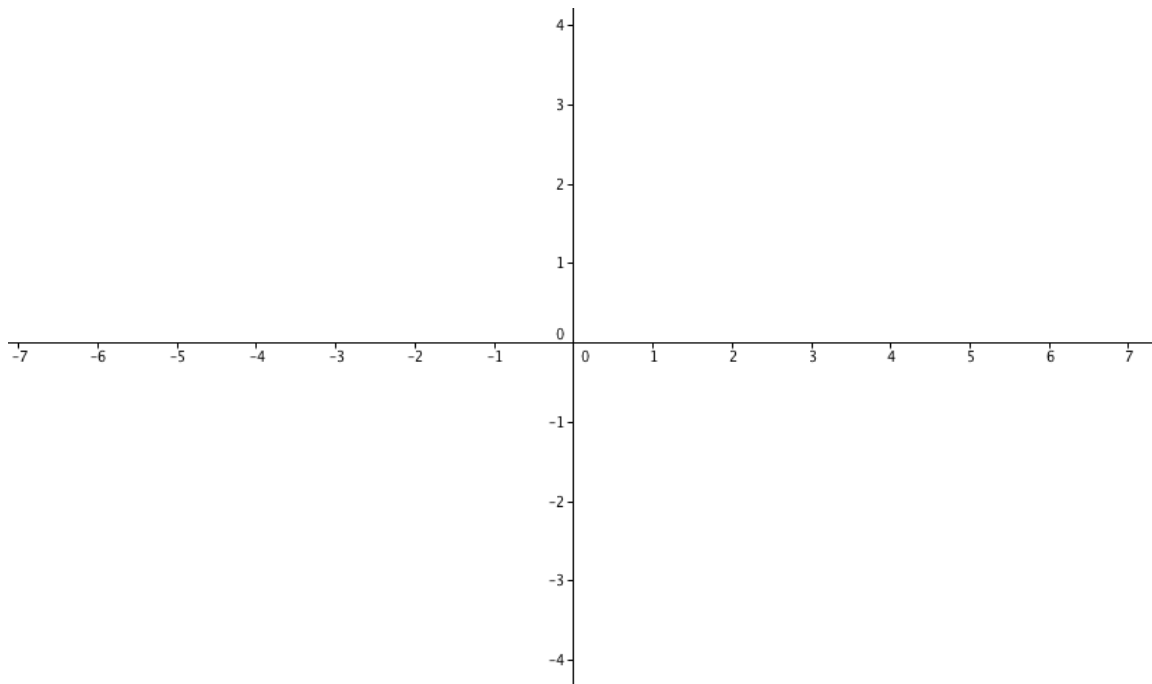


Will this formula correctly calculate the slope of  $f$  at the point  $x = 3$ ? Why or why not? If you think it does, explain why. If not, give a correct formula, making as few changes as possible to the one above. (Do not simply state a correct formula which is very different from this one.) Add details to the picture to help illustrate your answer.

3. The graph of a position function  $f(x)$  is shown on the top graph. Sketch a graph of the velocity graph on the bottom axes.



4. The graph of the function  $g(x)$  is pictured below.



Find these limits (write "DNE" if a limit doesn't exist):

$$\lim_{x \rightarrow 6} g(x) =$$

$$\lim_{x \rightarrow -4} g(x) =$$

$$g(4) =$$

$$\lim_{x \rightarrow -1^-} g(x) =$$

$$g(2) =$$

$$\lim_{x \rightarrow 3} g(x) =$$

$$\lim_{x \rightarrow 4} g(x+2) =$$

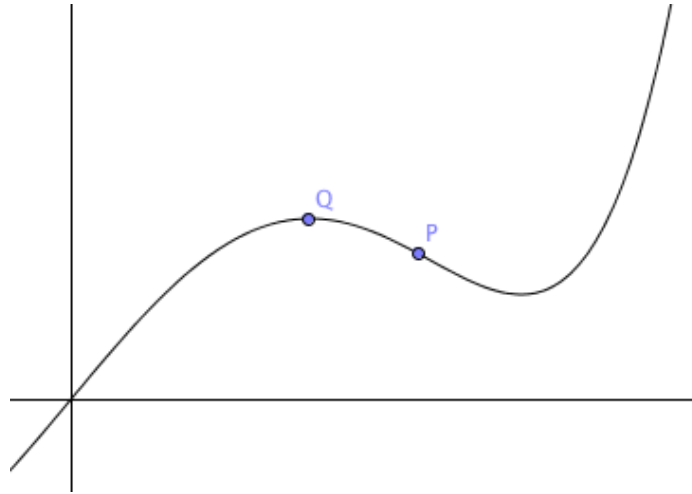
$$\lim_{x \rightarrow -1^-} g(x^2) =$$

5. Let  $f(x) = \frac{x^2 - x + 6}{x + 2}$ .

What is  $f(-2)$ ? Explain.

What is  $\lim_{x \rightarrow -2} f(x)$ ? Explain.

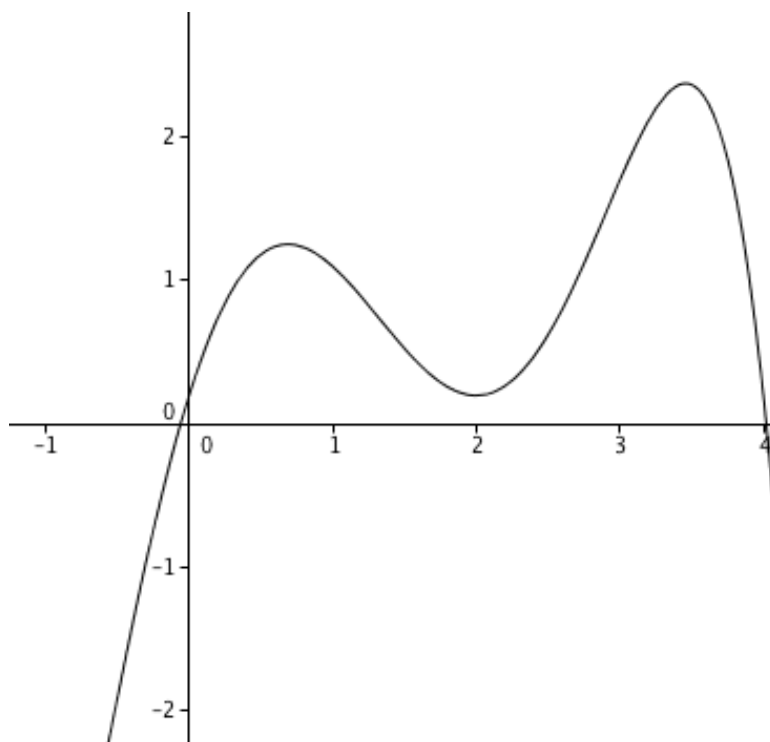
6. In the picture here, the point  $P$  is fixed and never moves. The point  $Q$  moves along the graph of the function toward  $P$  — getting arbitrarily close but never reaching  $P$ . Fill out the chart below (write “+” or “-”, in each row of the first column and “Incr.” or “Decr.” in each row of the last column.)



| Quantity in question                 | Positive or Negative? | Increasing or Decreasing? |
|--------------------------------------|-----------------------|---------------------------|
| $Q_x$ , the $x$ -coordinate of $Q$ : |                       |                           |
| $Q_y$ , the $y$ -coordinate of $Q$ : |                       |                           |
| distance from $P$ to $Q$ :           |                       |                           |
| $\Delta_x = P_x - Q_x$ :             |                       |                           |
| $\Delta_y = P_y - Q_y$ :             |                       |                           |
| $\frac{\Delta_y}{\Delta_x}$ :        |                       |                           |

7. What's your intended major? Why?

8. Here's a graph of the **derivative** of  $f$ :



This graph shows **the derivative of  $f$** ; that is, the graph shows  $f'(x)$ .

Answer the following questions about  $f$ ,  $f'$  and  $f''$ . Give **one point** between -1 and 4 (including the endpoints) where the statement is definitely true, using the scale on the graph to estimate the  $x$ -value. If there isn't enough information given to find such a point, write "Can't Tell".

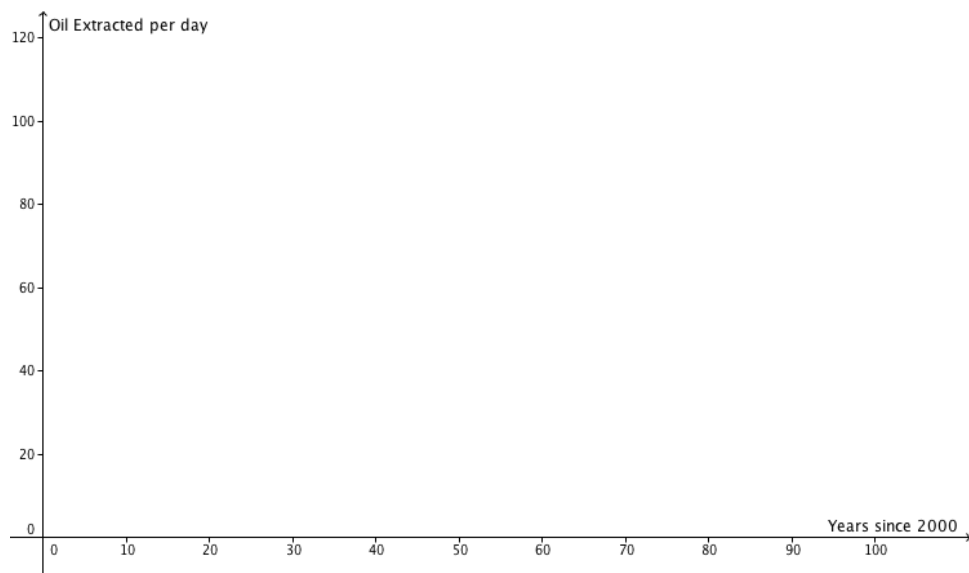
At what value(s) of  $x$  between -1 and 4 is ...

- a)  $f'(x)$  the greatest?
- b)  $f''(x)$  the smallest?
- c)  $f(x)$  the greatest?
- d)  $f(x)$  positive?
- e)  $f(x)$  concave down?
- f)  $f(x)$  the steepest?

9. Suppose  $f(x) = x^2 + 3$ . Using the definition of the derivative, show that the function that gives the instantaneous change of  $f$  is  $f'(x) = 2x$ . Show your work.

10. Let  $g(t)$  represent the amount of oil we expect to remove from below the earth's surface in year  $t$ . The units of  $g$  are in millions of barrels per day, and the horizontal axis ( $t$ ) is measured in years. Currently, we are removing about 93 millions of barrels per day — so  $g(2015) = 93$ . That number has been growing, but it's growth has been slowing down since 2000 (when we removed about 70 million barrels per day). According to many experts, that number will peak in 2030 (at approximately 100), then slowly fall, and then fall more quickly until 2070. After 2070, it will continue falling, but not as quickly.

Draw a sketch of what  $g(t)$  might look like on the given axes.



If we calculate the slope of  $g$ , what unit is the slope in?

Suppose the slope is currently 1.5 (in whatever units you described above). Explain what this means.

*Extra Credit:* The most populous country in North America is the United States. Give the most populous countries of Africa, Asia, Europe, and South America. (If a country is split between two continents, consider only the population that lives in the continent you are considering.)