Name: ____________________

Opportunity 3 Fall '14

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. Here is the graph of a function $f'(x)$.

Answer the following questions about related functions:

a) What are the critical points of f? Why?

b) Where does f have local minima? Why?

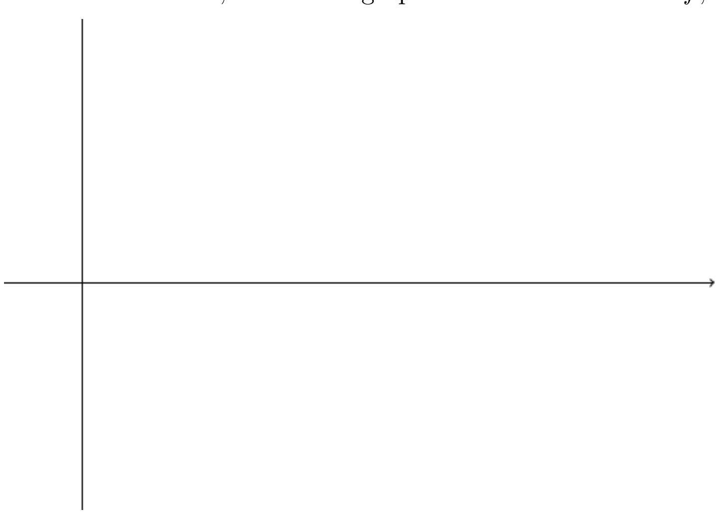
c) Where does f have inflection points (i.e. where f changes from concave up to concave down or from concave down to concave up.) Why?

2. What is the maximum value of the function $f(x) = x\sqrt{100 - x^2}$ on the interval $[0, 10]$?

3. Here's the graph of a function $g(t)$:

On the axes below, sketch the graph of the accumulation function $f(x)$, which represents the area under the function g between 0 and x:

On the axes below, sketch the graph of the **derivative** of f, or $f'(x)$:



If two of the three graphs above look similar, briefly explain why.

4. Fill in the blank with a statement about a function on the interval $(0, 1)$. Write a sentence explaining your reasoning. (The correct answer might "undetermined" meaning that the given information doesn't determine anything about the function in question.)

If f is positive, f' is ____________________________.

If f' is decreasing, then f is ____________________________.

If f is decreasing, then f' is ____________________________.

If f' is increasing, then f'' is ____________________________.

If f' is decreasing, then f is ____________________________.

5. The Fundamental Theorem of Calculus says, roughly, that the integral and the derivative undo each other. For each part below, give a more precise statement of that part of the FTC.

I. (The Derivative of an Integral is the original function.)

II. (The Integral of the Derivative is the original function*.)

6. You are erecting a tent, which will have a line of poles down the middle with the tent fabric going diagonally down to the ground. On each side of the poles, there is exactly 10 feet of fabric. You could have taller poles, but the tent would be narrow; you could have a wide tent, but you'd have to use very short poles. You'll be storing bags of rice in the tent, and you want to be able to put as much rice as possible inside the tent.

In this problem, you are asked to complete all the steps **except for the last one** to this optimization problem. That is, complete this problem to the point where you have a function you wish to optimize on a particular interval. Show your work.

7. Find these limits:

a) $\lim_{x\to\infty} \frac{4-6x+3x^4}{(x^3-2)(x+4)}$

b) $\lim x \to -\infty \frac{2x^2-100}{-10x+4}$

c) $\lim_{x\to 0} \frac{(2+x)^4-2^4}{x}$

d) $\lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$

8. Here's the graph of a function $f(x)$:

On the graph above, draw the boxes representing the Riemann Sum with eight boxes using right endpoints that would approximate the area under the function f between a and b.

Here's a formula that would calculate the area exactly:

$$\lim_{N\to\infty} \sum_{n=1}^{N} \Delta x_n \; f(x_n^*)$$

Explain each part of this formula and why, in its entirety, it calculates the exact area under the function f between a and b.

Extra Credit: The countries of the world vary widely in what proportion of their citizens are behind bars. Of the countries with at least a million people, name the four with the highest incarceration rates. (You'll earn half a point each for correct answers.)