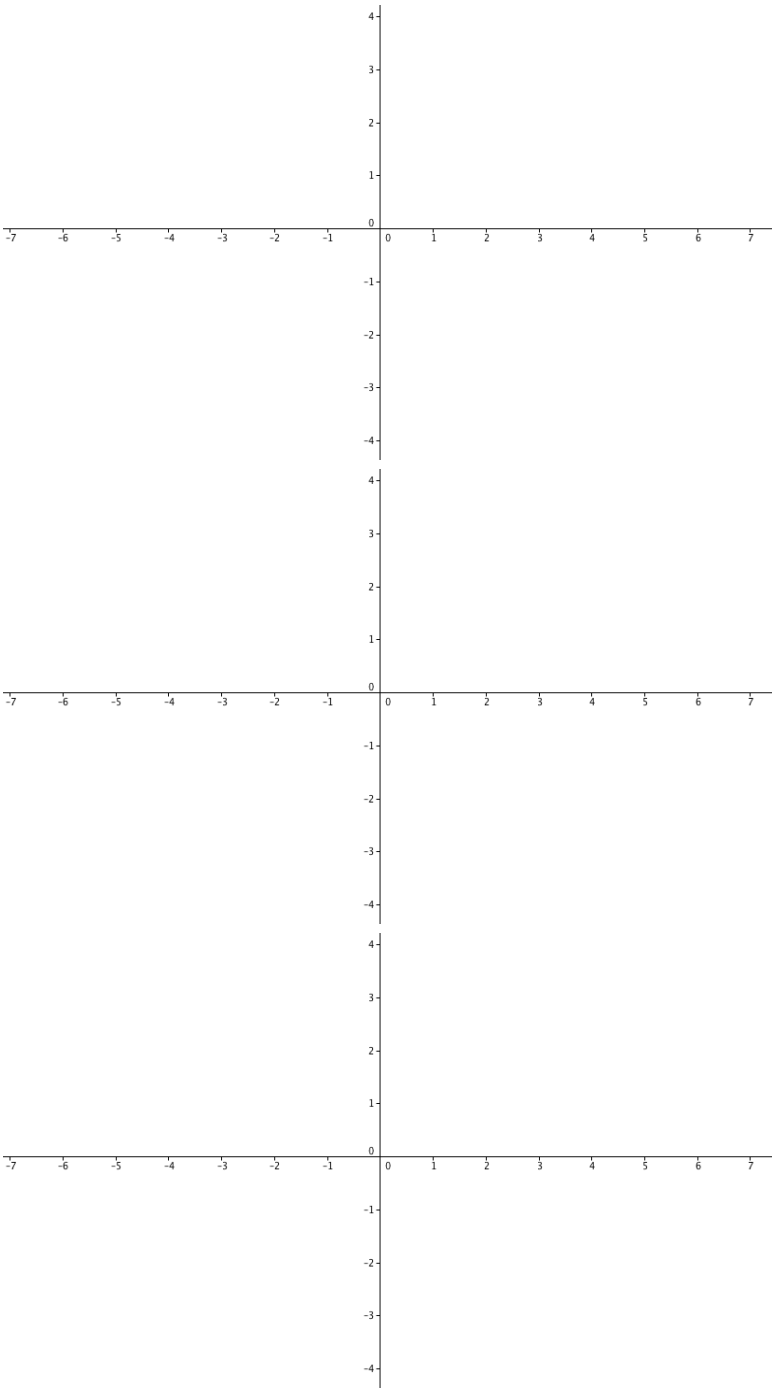
Name: _____

Opportunity II

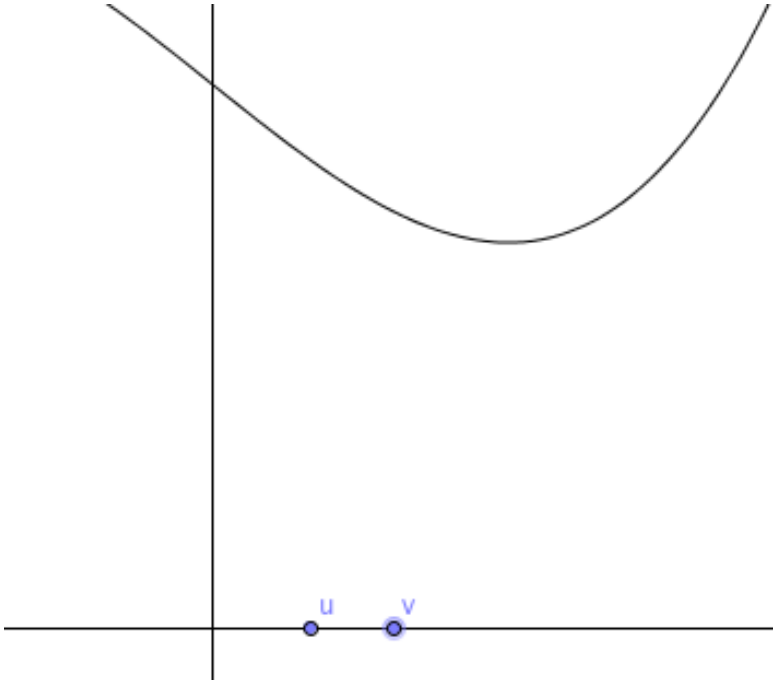
Fall '14

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. The graph of a position function $f(x)$ is shown on the bottom graph. Sketch the velocity graph on the middle axes. Sketch the acceleration graph on the upper axes.



2. The graph below depicts a function f and two points on the x-axis, named u and v.



Put the following four values in order from least to greatest: $f(u)$, $f(v)$, $f'(u)$, and $f'(v)$:

$$\leq \qquad \leq \qquad \leq$$

Using only the given symbols (that is, without introducing any new variables), write a correct **definition** of the slope of f at v (a.k.a. $f'(v)$.)

Briefly explain why your formula works.

3. Using the derivatives of $\sin(x)$ and $\cos(x)$, prove that $(\sec x)' = \sec x \tan x$.

4. In each case below, find y'.

a) $y = \sin(x^4 + 1)$

b) $y = f[g(\sqrt{x}) + g(x)]$

c) $y^2 \sin x + xy = \tan x$

5. What is your major? Why?

6. Finish the statement of the Mean Value Theorem:

If f is continuous on the interval $[a, b]$ and differentiable on the interval (a, b), then there exists a point $c \in (a, b)$ where....

Draw a picture that explains the Mean Value Theorem - be sure to label all points so that every part of your formula above is shown.

7. Using the definition of the derivative that has $h \to 0$, write out a limit that would give the slope of the function $f(x) = \tan(\frac{x^3}{1+x})$ at the point $x = 1$. Please, please do not try to simplify this limit.

Find $f'(x)$ using the various derivative rules.

8. Let $f(x) = x^3 - 3x^2 - 24x$.

Find all critical points of f.

On what interval(s) is f increasing?

On what interval(s) is f decreasing.

Extra Credit: Of the (roughly) 300 million people currently in the United States, four have tested positive for Ebola and are in treatment. For 1 point each, name the three states where these four people are currently being treated. (For comparison, this year in West Africa there have been about 10,000 cases and 5,000 deaths.)