Name: _____

Opportunity I

Fall '14

Yay!! An Opportunity! So exciting!

No calculators or cell phones are allowed — please turn them off and zip them away in your bookbag. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available, but you should only turn in the exam.

1. In class we described a limit as "the end result of an infinite process."

Describe in your own words what this means.

Explain what this idea of the limit has to do with finding the instantaneous change of a function:

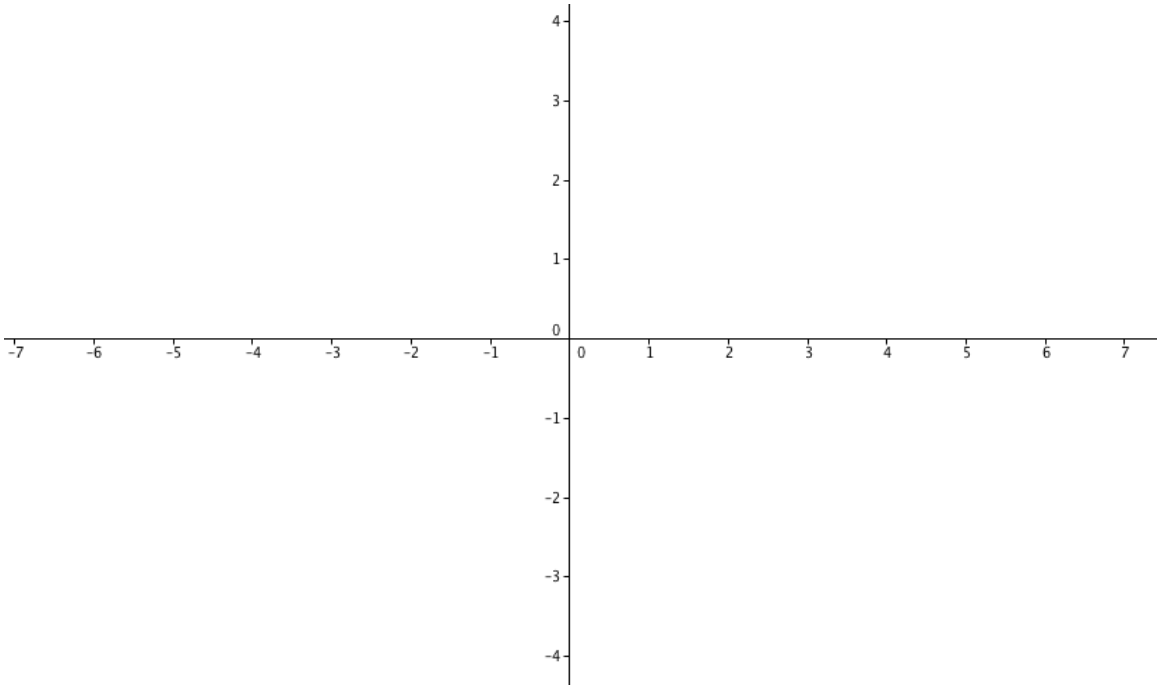
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

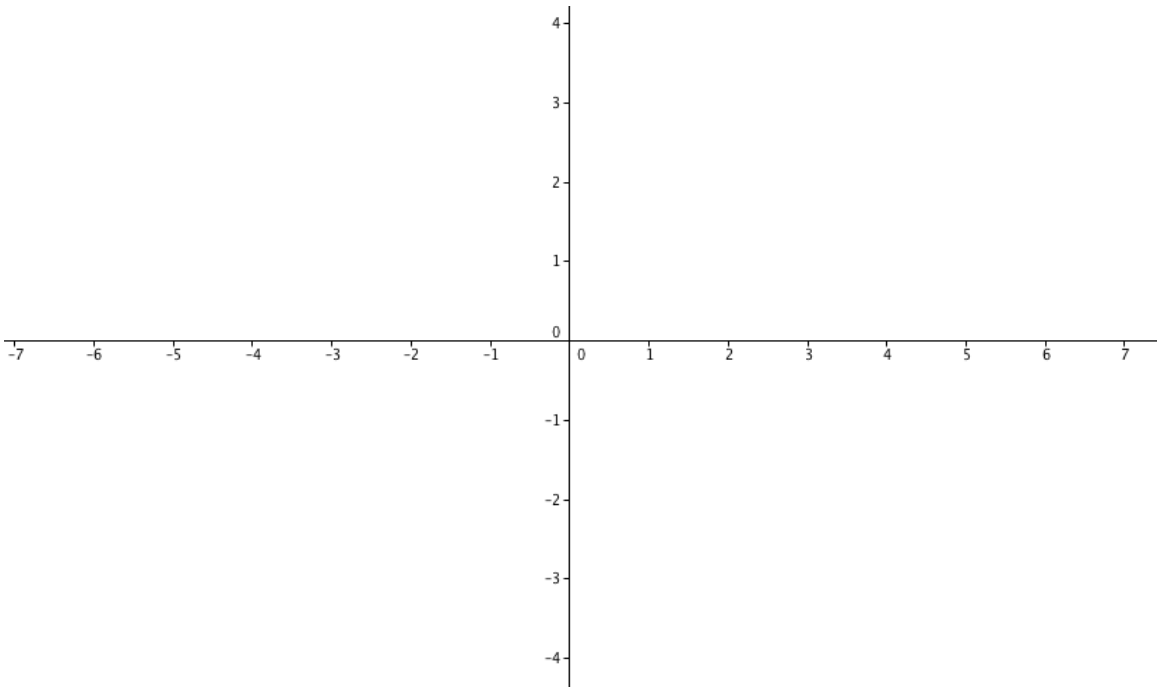
2. Let

$$h(x) = \begin{cases} \frac{2x^2-4x-6}{x-3} & \text{if } x \neq 3 \\ ? & \text{if } x = 3. \end{cases}$$

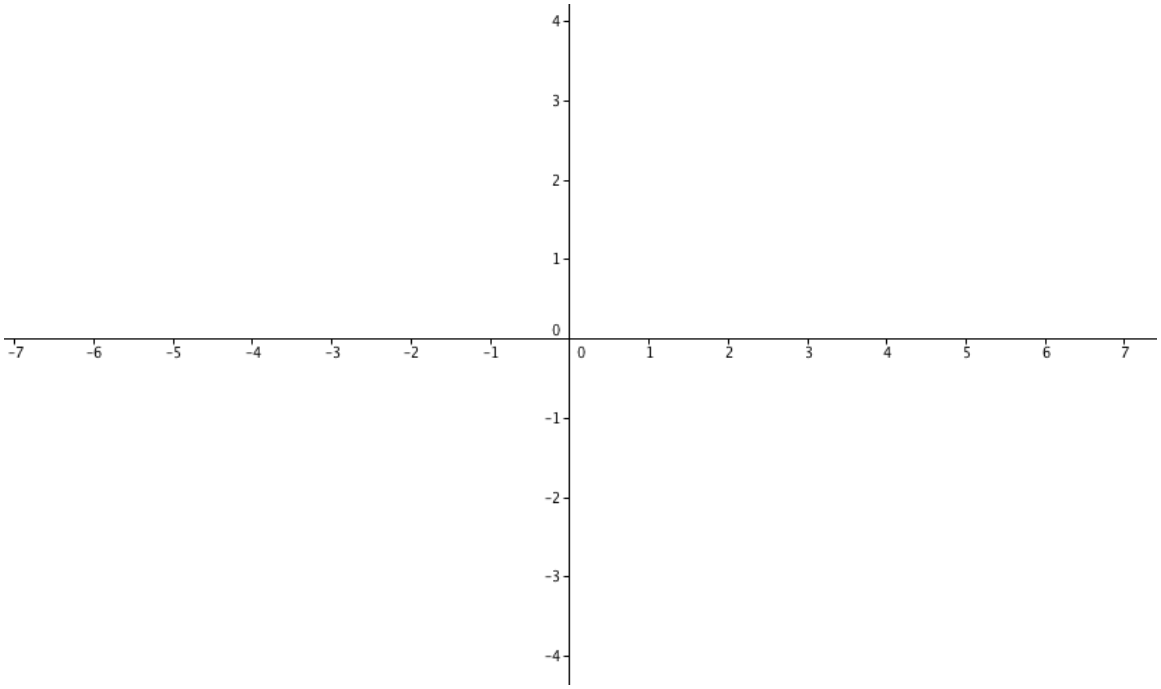
What should you choose for the value of $h(3)$ so that h is a continuous function for all x? Why?

3. The graph of a position function $f(x)$ is shown on the bottom graph. Sketch a graph of the velocity graph on the upper axes.





4. The graph of the function $g(x)$ is pictured below.



Find these limits (write "DNE" if a limit doesn't exist):

$\lim\limits_{x\to -1} g(x) =$ $\qquad$ $\lim\limits_{x\to 3^+} g(x) =$ $\qquad$ $g(4) =$

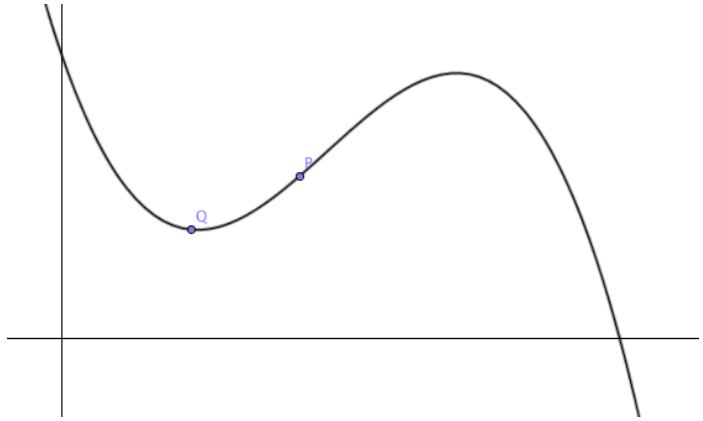
$\lim_{x\to 3^-} g(x) =$ $\qquad$ $g(-2) =$ $\qquad$ $\lim\limits_{x\to 3} g(x) =$

$\lim\limits_{x\to 1^+} g(x+2) =$ $\qquad$ $\lim\limits_{x\to -1^-} g(x^2) =$ $\qquad$ $\lim\limits_{x\to 2} x\, g(x) =$

5. Returning to the function g in the previous problem, for each point where g is discontinuous, list the point and the properties of that discontinuity (jump, removable, infinite).

Point (x-value)	Type of Discontinuity	Continuous from Left?(Y/N)	Continuous from Right? (Y/N)

6. In the picture here, the point P is fixed and never moves. The point Q moves along the graph of the function. For each of the following, describe in a word or several words what happens as Q moves toward P along the curve — getting arbitrarily close but never reaching P. (For instance, an answer might be "it's negative, and decreases to $-\infty$ in the limit.")



Q_x, the x-coordinate of Q:

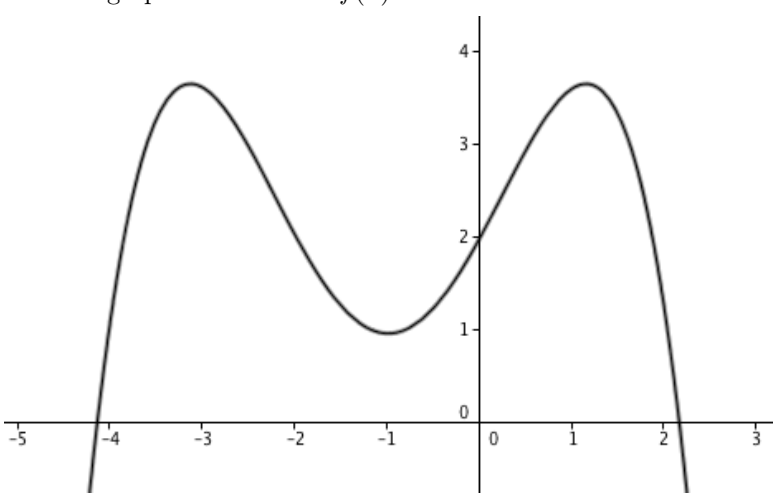
Q_y, the y-coordinate of Q:

$\Delta_x = P_x - Q_x$:

$\Delta_y = P_y - Q_y$:

$\frac{\Delta_y}{\Delta_x}$:

7. Here's a graph of a function $f(x)$:



Put the following quantities in order from least to greatest:

$$f'(-2), f'(-1), f'(0), f'(1), f'(2)$$

$$\leq \qquad \leq \qquad \leq \qquad \leq$$

Briefly explain your answers.

8. What's your favorite non-academic thing to do? Why?

9. Consider the formula $\lim_{h\to 0} \frac{f(5)-f(5-h)}{h}$ and the corresponding graph

Will this formula correctly calculate the slope of f at the point $t = 5$? Why or why not? If you think it does, explain why. If not, make an adjustment to the formula so that it is correct. (Do not simply state a correct formula which is very different from this one.) Add details to the picture to help illustrate your answer.

10. Using the definition, calculate the instantaneous change of the function $f(x) = \sqrt{x+4}$ at $x = 5$. Show your work. (Note to those who have had Calculus before: You will receive no credit for simply taking the derivative and plugging in $x = 5$.)

11. Find the following limits:

$$\lim_{x \to 0} \frac{\sin(x)}{2x}$$

$$\lim_{x \to -2} \frac{x^2 + 2x - 8}{x - 2}$$

$$\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1}$$

Extra Credit: Last weekend, around 300,000 people marched in New York City urging world leaders to combat climate change. The U.S. is one of the four worst countries in terms of greenhouse gasses emitted per person. Name the other three.