FINAL OPPORTUNITY   FALL '05

No calculators or cell phones are allowed. If you have any questions, please ask Simon (or Dave, when he gets here). Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. Integrate the following using the Fundamental Theorem of Calculus.

a) $\displaystyle\int_0^\pi 2\sin x\ dx$
b) $\displaystyle\int_0^2 x^{1987}\ dx$

2. Find y':

a) $y = \cos x \sin x$
b) $y = \sqrt{x} + x^5$

c) $y = \frac{\tan x}{\sqrt{x+1}}$
d) $yx^2 - y^2\sin x = \sec x$

3. Write the definition of the following integral using n-equally spaced intervals and right endpoints. Explain each part of your forumla.

$$\int_1^3 4x^5 - x^3\ dx$$

Find the value of this integral using the Fundamental Theorem of Calculus.

4. Suppose $s(t)$ is Ruby's position along a line at time t, and that she starts out at $x = 1$. Also, suppose her velocity is given here (in the middle graph). Sketch her position function $s(t)$ on the top graph and her acceleration $a(t)$ on the bottom graph.

5. Let $g(x) = \displaystyle\int_x^3 \frac{1}{1+t^2}\ dt$.

For x's between 0 and 3, is g increasing or decreasing? Why?

Find $g'(x)$. Explain your reasoning.

Sketch $g(x)$ on the axes given.

6. Throughout this semester we've used the idea of limits to approximate various things. We discussed this idea at a naive level, as well as a more sophisticated one. At that deeper level, the various uses of the limit were characterized by different ways of filling in the following chart:

You can make		as close as you want to		by making		sufficiently	
	_____		_____		_____		_____ .

For instance, the very first limit we looked at involved me walking toward the door, with step lengths that were being cut in half each time. This would be described in the above chart by filling in the blanks as follows:

You can make **Dave** as close as you want to **the door** by making **the number of steps** sufficiently **large**.

Fill in this chart for the derivative. Explain your answer.

Fill in this chart for the integral. Explain your answer.

7. You probably studied for hours for this final, and much of what you studied didn't appear on it. Write a problem you thought should be on the final and solve it. (You may not use problems which appeared on earlier opportunities.) You will be graded both on the writing of the problem and your solution.

Extra Credit: Name the scales on which the severity of each of the following natural disasters are measured: tornados, hurricanes, earthquakes, and asteroid impacts.