

Name: _____

OPPORTUNITY $-2e^{i\pi} + \lim_{x \rightarrow \infty} \frac{x^2}{x^2+10}$

Fall '05

No calculators or cell phones are allowed. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. Fill in the blank. Write a sentence explaining your reasoning. (The correct answer might "undetermined" meaning that the given information doesn't determine anything about the function in question.)

If f'' is positive, f' is _____.

If f' is increasing, then f is _____.

If f is decreasing, then f'' is _____.

Here is the graph of a function $f'(x)$.

Answer the following questions about related functions:

- What are the critical points of f ? Why?
- Where does f have local maxima? Why?
- Where does f have inflection points? Why?

2. Find the following two limits. Show your precise mathematical work (not just how you find the answer in your brain.)

a) $\lim_{x \rightarrow -\infty} \frac{x^3 + \pi x^2 - 10^{10}}{-2x^4 - 2x^2}$

b) $\lim_{x \rightarrow \infty} \frac{(x-2)(2x^2 - \pi)}{x(500 - x^2)}$

3. On the lower axes is a graph of a function $f(x)$. On the top axes, draw a possible function for the antiderivative of f , namely $F(x)$ (so that the graph of $f(x)$ is also the graph of $F'(x)$). You may assume that $F(x)$ is continuous.

4. Find all anti-derivatives for the following functions:

a) $f(x) = x^5 - \cos x$

b) $g(x) = \sqrt{x}$

c) $h(x) = x^\pi$

d) What do you want to be when you grown up?

5. Let $f(x) = 2x^2 - 1$.

Find the approximation to the area under f between $x = 1$ and $x = 2$ using 4 boxes and left end-points. Show your work. (You do not need to simplify.)

Give an expression for the area under f between $x = 1$ and $x = 2$ using n boxes and right endpoints. Use summation notation to express your answer. Show your work. (You do not need to find the answer.)

6. **A new running track!** In case you didn't know, our running track sucks. It's made of material hard enough to kill your knees. So let's just fantasize for a minute...

Design a new track for the college. Like all tracks, it will be 400 meters long with two straightaways on the sides and two semi-circles connecting them. Because our Lacrosse teams need maximum space to play on the inside of the track, let's try to maximize the amount of grass on the inside of the track.

Find out what dimensions will maximize the area inside the track. Be sure to justify why your dimensions give the biggest area. If you would like a numerical approximation to any values, ask Dave.

7. Use your knowledge of Calculus to sketch an accurate graph of the following function.

$$g(x) = \frac{x - 1}{x^2}$$

Show your work.

Extra Credit: President Bush is finishing up a trip to Asia during which he visited four countries. In addition to China and Japan, what two countries did Bush visit?