

OPPORTUNITY 3 + $2e^{i\pi}$

Fall '05

The excitement builds...the crowd goes wild...the Calculus students enter the classroom in their first chance to show what they've learned!

No calculators or cell phones are allowed. If you have any questions, please ask Simon or Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. Say the following out loud: "I am smart and I can do this."

Really. I wasn't kidding. Say it out loud now.

2. Here is a sketch of the graph of $y = x^3 - x$. On the axes below, sketch a graph of $y' = (x^3 - x)'$. Be sure to match up important points of y with the appropriate points on y' . Write a few sentences which explain how you arrived at your graph.

Algebraically calculate y' . Does your answer match your graph?

3. The graph of the function $g(x)$ is picture below.

Answer the following - explain each answer briefly.

a) $\lim_{x \rightarrow -3} g(x) =$

b) $g(-3) =$

c) $\lim_{x \rightarrow 2^-} g(x) =$

d) $\lim_{x \rightarrow 2} g(x) =$

e) $\lim_{x \rightarrow -1} g(x+1) =$

f) $\lim_{x \rightarrow -1} g(x)g(x+1) =$

4. Let $f(x) = 2e^{i\pi}x^2 = -2x^2$.

Find the equation of the tangent line to f at the point $(1, -2)$. Explain every step of your calculation.

5. Find the following limits. Give reasons for your answers.

$$\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2}$$

$$\lim_{x \rightarrow 2} \frac{x^2+4x+3}{x^2-4x+4}$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x}$$

One of the above limits is the definition of a derivative. Which one? What function is being differentiated? At what point?

6. Suppose that $D(t)$ measure the depth of the water down at the dock as a function of time (measured in minutes), and is measured in meters.

What units is $D'(t)$ measured in? Explain.

What does it mean if $D'(t) > 0$?

Under normal conditions, D varies between 3 and 4 meters. During Hurricane Isabel, $D(t)$ reached 7 meters. What does the Intermediate Value Theorem tell you about $D(t)$?

7. Consider the mathematical expression:

$$\lim_{x \rightarrow \pi} f(x) = 2$$

Explain what this means in naive terms.

Explain what this means in the least naive terms you can.

8. Consider the following formula:

$$\lim_{h \rightarrow 0} \frac{f(a - h) - f(a)}{-h}$$

Will this formula correctly calculate the slope of f at the point $(a, f(a))$? Why or why not? Draw a picture to illustrate your answer, and explain it in a short paragraph.

Extra Credit: Why do hurricanes in the Northern Hemisphere rotate counter-clockwise?