Calculus I, Fall '04 Name:_____

Final Exam

You're almost done with Calc I! Yippee!

As before, no calculators, cell phones, or spare brains. Please show your work for *all* problems. As you might have expected, this Calculus Final will require multiple uses of The Fundamental Theorem of Calculus. Please note every time you use it on this exam - write "FTC I" or "FTC II" to indicate which part of the theorem you are using.

1. Calculate the following integrals:

$\int 1986x^{1987} + 1988x \, dx$ $\qquad$ $\int_1^{\pi} \pi^{-1} \, x^{\pi-1} \, dx$

$\int \sin(x)\cos(x) \, dx$ $\qquad$ $\int_1^2 x\sqrt{1+x^2} \, dx$

2. A fellow student is trying to write out a Riemann Sum which would calculate the area under the $\sec^2 x$ function between $x = 2$ and $x = 4$. She writes the following:

$$\lim_{N\to\infty} \sum_{i=1}^{N} \sec^2(2 + \frac{i}{N})\frac{2i}{N}$$

Is her sum correct or not? If you think it is correct, say why. If you think it is incorrect, say why and give a correct formula.

3. For each of the following, find y'.

a) $y = (1 + x^2)^{3/2}$

b) $y = (1 + x^{1/3})^{2/3}$

c) $\cos(xy) = 3x^2 + y^3$

4. What's the derivative of $f(x) = \int_0^x \frac{1}{1+t^2} \, dt$? Explain.

What's the derivative of $g(x) = \int_0^{x^2} \frac{1}{1+t^2} \, dt$? Explain.

5. Use your knowledge of Calculus to graph $g(x) = \frac{4}{x^2-2x-3}$. Show your work.

6. The one fundamental idea underlying all of Calculus is the limit. Find the following limits - explain your reasoning for each.

a) $\lim_{h\to 0} \dfrac{\int_x^{x+h} t^2 + \sin t \, dt}{h}$

b) $\lim_{x\to\infty} \dfrac{(x-\pi)^3}{\pi x^3 - \pi^2 x^2 - \pi^3}$

c) $\lim_{x\to 0} \dfrac{\cos x - 1}{x}$

7. On the middle axes is a graph of the function $g(t)$. On the top axes graph $\int_0^x g(t)dt$. On the bottom axes graph $g'(t)$.

Extra Credit: In 1937, one country invaded another starting World War II. Name both countries. Bonus points for naming the leaders of each country at that time.