Calculus I, Fall '04   Name:_____

Opportunity III

You know the drill. No calculators, no cell phones, no looking at someone else's paper, no throwing things at Jeff; lots of explaining your answers, lots of writing clearly, lots of writing on scratch paper and then showing your work on the exam.

1. Find the following limits:

a) $\lim\limits_{x \to \infty} \dfrac{\pi^\pi x^2 - 100x + \pi}{(x-1)(x-3)}$

b) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

c) $\lim\limits_{x \to \infty} \dfrac{1 + \sin(2x)}{x + 2}$

2. If possible, draw a function which has *all* of the following properties. If not possible, say why not and draw a graph with as many of the properties as possible.

a) f has exactly two x-intercepts and a negative y-intercept.

b) f has vertical asymptotes at -2 and π and horizontal asymptotes at 0 and 3.

c) f has critical points at $x = -1$ and $x = 1$.

d) f is concave up on $(-\infty, -2)$, $(0, \pi)$, and (π, ∞).

e) f is concave down on $(-2, 0)$.

3. Here is the graph of $f'(x)$. Below the graph, give the points or set of points which satisfy the given property. ("No points" or "not enough information" are possible answers.) Give a brief explanation for each.

a) f has a local min:

b) f has a local max:

c) f is concave down:

d) Inflection points of f:

e) $f''(x) > 0$:

4. For each of the following, find y':

a) $y = \sec(x^2 + 1)$ b) $x^2 y = \sin(2y)$.

5. Here is the graph of $g''(x)$. Below the graph, give the points or set of points which satisfy the given property. ("No points" or "not enough information" are possible answers.) Give a brief explanation for each.

a) g is concave up:

b) Inflection points of g:

c) $g'(x)$ is increasing:

d) g has a local maximum:

6. Draw something interesting here.

7. You are designing the shot put area for the St. Mary's Olympic Stadium. By IOC regulations, it must be the shape of a sector of a circle with the radius at least 25 meters (since the current world record is just over 23 meters). You have enough sand to cover 900 square meters of area. To keep drunken college students out, the shot put area will be surrounded by a fence. Assuming you decide to use all the sand (so that the area covered is exactly 900 square meters, what dimensions should the shot put area be to minimize the cost of the fence? (Show all of your work.)

Extra Credit: Name the five most populous cities in the world (actual city population, not metropolitan-area population).