

Opportunity II

Say this out loud: "I can do Calculus!" Say it again - this time like you really mean it.

You know the drill. No calculators, no cell phones, no looking at someone else's paper; lots of explaining your answers, lots of writing clearly, lots of taking your time to think through problems and answering correctly.

1. For each of the following, find y' :

$$y = \sin(x^2)$$

$$y = \sqrt{1 + \sqrt{1 + \sqrt{x}}}$$

$$y = \csc \pi x \tan \pi x$$

$$y = \sec^2 x - \tan^2 x$$

$$y \sin(y) + xy = \pi^\pi y$$

$$y = x^\pi + \csc x$$

2. Given the following information about f , answer the questions below:

x	0	1	2
$f(x)$	1	-2	3
$f'(x)$	100	4	π

- If $g(x) = (f(x))^2$, find $g'(2)$ if possible. Explain.
- If $h(x) = f(f(x))$, find $h'(0)$ if possible. Explain.
- Find $\lim_{x \rightarrow 1} f(x)$ if possible. Explain.

3. For each expression below, find $y''(x)$.

a) $y = x^4 \sin x$

b) $x^2 - 2y^2 = 1$

4. A commercial jet aircraft needs to reach a speed of about 50 meters per second before taking off (that's about 120 mph). On the runway, after releasing its brakes, the distance it covers (measured from the beginning of the runway) is roughly $D(t) = \frac{2}{3}t^2$, where t is measured in seconds and D is measured in meters.

- How fast is the airplane going 15 seconds after it starts? Explain.
- How long does it take for this airplane to take off? Explain.
- How much of the runway does it use before taking off? Explain.

5. True or False? If True, explain why. If False, give a counterexample.

- If $g(x)$ has an absolute maximum at $x = \pi$ and g is continuous for all x in the interval $(1, 4)$, then $g'(\pi) = 0$.
- If $h(x)$ is continuous on $(3, 5)$ and differentiable on $[3, 5]$, then there is a point $c \in (3, 5)$ where

$$h'(c) = \frac{h(5) - h(3)}{5 - 3}$$

- If $f(x)$ is continuous on $[0, 4]$ then f has an absolute maximum and minimum on $[0, 4]$.

6. Find the absolute maximum, absolute minimum, local maxima, and local minima of the function $f(x) = \sin x + \cos x$ on the interval $[0, \pi/2]$.

7. During a recent hurricane, the storm surge pushed lots of ocean water into the Chesapeake Bay and up the St. Mary's River. The salinity in the River (measured in ppt - parts of salt per thousand) rose from 10 ppt at 6am to 20 ppt at noon. What does the Mean Value Theorem tell you about the salinity?

Extra Credit: 'Tis the Season - Name the last 4 major party candidates to lose the Presidential election.