Calculus I, Fall '04
Name:_____

Opportunity I

Yippee! It's your first chance to show me what you've learned so far this semester. No calculators or cell phones are allowed. If you have any questions, please ask me or Jeff. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. For what values of x is $|x - 3| > 4$?

Let $f(x) = 1 - x^2$ and $g(x) = \frac{1}{x}$.

Find $g \circ f(x) = g(f(x))$ and state its domain.

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2. Explain how limits and derivatives are related. (A couple of sentences should suffice.)

3. The graph of the function $g(x)$ is picture below.

Answer the following.

$$\lim_{x \to -5^+} g(x) = \qquad\qquad \lim_{x \to -2^+} g(x) = \qquad\qquad \lim_{x \to 4^-} g(x)$$
$$\lim_{x \to 0} g(x) = \qquad\qquad\qquad g(0) = \qquad\qquad\qquad \lim_{x \to 2} g(x) =$$
$$\lim_{x \to -2} g(x + 2) = \qquad\qquad \lim_{x \to -2^-} g(x^2) =$$

4. Let
$$f(x) = \begin{cases} x^2 + 1 & \text{if } x > 1 \\ ax + b & \text{if } -1 \le x \le 1 \\ -2x^2 - 2 & \text{if } x < -1 \end{cases}$$
Choose values for a and b so that $f(x)$ is continuous for all x. Explain your reasoning.

5. Find the following limits. Give reasons for your answers.

$$\lim_{x \to \frac{3\pi}{4}} \frac{\tan(x)}{x - 2}$$

$$\lim_{x \to -2} \frac{x^2 + 2x - 8}{x - 2}$$

$$\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1}$$

One of the above limits is the definition of a derivative. Which one? What function is being differentiated? At what point?

6. Let $g(x) = \sqrt{2x}$. Use the definition of the derivative to find $g'(a)$. (Here you must calculate the limit.)

7. Consider the following formula:

$$\lim_{t \to 5} \frac{f(5) - f(t)}{t - 5}.$$

Will this formula correctly calculate the slope of f at the point $x = 5$? Why or why not? If you think it does, explain why. If not, make an adjustment to the formula so that it is correct. (Do not simply state a correct formula which is very different from this one.) Add details to the above picture to help illustrate your answer.

8. The graph of $j(x)$ is shown on the top axis. Sketch a graph of $j'(x)$ on the lower axis being as accurate as possible.

Extra Credit: At what time does every full moon rise?