Calculus I

Name:_____

Opportunity III

Here you go! This is your last chance before the final to show me what you've learned this semester. No calculators or cell phones are allowed. If you would like the decimal approximation of any numbers, come see me. If you have any questions, please ask me. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. Find y''.

$$y = x\sec^2 x - x\tan^2 x$$
$$y = \sin(x^2)$$
$$y^2 + xy = \cos(x) \text{ (Leave your answer in terms of } x, y, \text{ and } y'.\text{)}$$

2. Calculate the following limits:

$$\lim_{x\to\infty} \frac{x + \sqrt{x^2 - 1}}{3x + e^{\sqrt{\pi}}}$$
$$\lim_{x\to 2} \frac{x^2 - x - 2}{x^2 + x - 6}$$
$$\lim_{x\to\infty} \frac{4(x+1)(x-2)}{7x^2 - 2x + \pi}$$

3. State the Mean Value Theorem. Be sure to include the hypotheses.

At Thanksgiving dinner last year, I sat down at the table weighing 180 pounds. Three hours later, I stumbled off to the couch weighing 186 pounds. What does the Mean Value Theorem say about my weight? Explain.

4. On the middle set of axes is the graph of the function $f'(x)$. Sketch the functions $f(x)$ on the top axes and $f''(x)$ on the bottom axes.

5. Matt likes to fly kites. On particularly windy day (when he and his roommate managed to wake up), he let out 30 meters of line, and Dave let the kite go. The wind was so strong that it flew straight up at a rate of 1 meter per second. When it was 40 meters off the ground, how quickly was Matt letting out the line? Show your work.

6. Use all available tools to graph the function $f(x) = \frac{1-x^2}{1+x^2}$. Label all maxima, minima, asymptotes, inflection points, and show all of your work.

7. The U.S. Postal service will accept rectangular boxes only if the "length + width around largest point" is less than 130 inches. The "width around the largest part", sometimes called the girth, is the largest perimeter of the box, measured in the directions perpendicular to the length. Find the rectangular box of largest volume which would be allowed under this rule. (You may assume that the width and height of the largest box are equal.)

Extra Credit: Ten people have declared their intention to run for the Presidency of the United States next fall. You get half a point for each one you can name.