Calculus I   Name:_____

Opportunity II

Here you go! This is your chance to show me what you've learned so far this semester. No calculators or cell phones are allowed. If you have any questions, please ask me. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. For each expression below, calculate y'.

$y = \frac{r^2}{1+\sqrt{r}}$ $\qquad\qquad\qquad\qquad\qquad$ $y = \sqrt{x + \sqrt{x}}$

$y = x \sec x$ $\qquad\qquad\qquad\qquad\qquad\quad$ $y = \frac{e^{\sqrt{\pi}}}{(1-x)^{3/2}}$

$y = \sin^2(x)$ $\qquad\qquad\qquad\qquad\qquad\quad$ $y = \sin(x^2)$

2. The graph of a function $f(x)$ is shown below. On the second set of axes, sketch a graph of $f'(x)$. Be as accurate as possible.

3. Prove the Sum Rule for derivatives, that is, show that $(f + g)' = f' + g'$. Hint: Use the definition of the derivative three times.

4. Find y' for each of the following functions:

$y = \frac{x^2}{x^{1/3}(tan x)}$

$y = \frac{\sin(\cos x)}{x}$

$y^3 + 2xy - x^2 y^2 = \sin(\pi/4)$

$xy + \sin^2(x)\sin(2x) = e^{\sqrt{\pi}} x$

5. Suppose that $f(x)$ is a positive, decreasing function which is differentiable for all x, and $g(x) = -x^2 + 1$. Let $h(x) = f(g(x)) = f \circ g(x)$ and $j(x) = g(f(x)) = g \circ f(x)$.

Is $h(1)$ positive or negative? Why?

Is $h'(1)$ positive or negative? Why?

Is $j'(-1)$ positive or negative? Why?

6. My dog Piper runs along a line at home during the day. One day, while watching her from the deck, I saw her run along the line so that her position (when measured from the middle of the line) was

$$s(t) = t^3 - 3t^2 - 9t,$$

where s is measured in meters and t is measured in seconds. After following this function for 5 seconds, she laid down in the grass exhaustedly.

a) What was her initial velocity?

b) For what values of t was she moving to the left?

c) When was she traveling the fastest?

d) What was her average velocity over those 5 seconds?

Extra Credit: For one point each, name the last five recipients of the Nobel Peace Prize (in any order).