

## Opportunity I

Here you go! This is your chance to show me what you've learned so far this semester. No calculators or cell phones are allowed. If you have any questions, please ask me. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. Let  $f(x) = \sqrt{1+x}$  and  $g(x) = \sqrt{1-x}$ .

State the domain of  $f$  and the domain of  $g$ .

Find  $h(x) = f(x) + g(x)$  and state its domain.

Find  $j(x) = \frac{f(x)}{g(x)}$  and state its domain.

Find  $k(x) = f(g(x))$  and state its domain.

2. The graph of the function  $g(x)$  is picture below.

$$\lim_{x \rightarrow -4} g(x) = \quad \lim_{x \rightarrow -2^+} g(x) = \quad g(4) =$$

$$\lim_{x \rightarrow 2^-} g(x) = \quad g(-2) = \quad \lim_{x \rightarrow 4} g(x) =$$

$$\lim_{x \rightarrow 2^+} g(x+2) = \quad \lim_{x \rightarrow -2^-} g(x^2) = \quad \lim_{x \rightarrow 2} x g(x) =$$

List all the points where  $g$  is discontinuous.

3. Let

$$f(x) = \begin{cases} 2x & \text{if } x \leq 1 \\ \sqrt{x+3} & \text{if } 1 < x < 6 \\ (x-4)^2 & \text{if } x \geq 6 \end{cases}$$

Is  $f$  continuous at  $x = 1$ ?  $x = 6$ ? Why or why not?

4. Find the following limits. Give reasons for your answers.

$$\lim_{x \rightarrow \pi/4} \frac{\sin x}{x-1}$$

$$\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x-2}$$

$$\lim_{x \rightarrow 1^-} \frac{x^2 - 2x - 3}{x-1}$$

5. Consider the following formula:

$$\lim_{h \rightarrow 0} \frac{f(3+h) - f(3-h)}{2h}$$

Will this formula correctly calculate the slope of  $f$  at the point  $x = 3$ ? Why or why not? (You should write 3 or 4 sentences, referring to the picture as necessary.)

6. Let  $h(x) = \tan x$ . Write the definition of  $h'(\pi/3)$ , the instantaneous change in  $h$  at the point  $x = \pi/3$ . Simplify where possible. (You do not need to calculate the limit, just write it down.)

Let  $g(x) = 2x^2 - 1$ . Use the definition of the derivative to find  $g'(a)$ . (Here you must calculate the limit.) Graph both  $g$  and  $g'$  on the given axes.

**Extra Credit:** For one point each, name an artist from each of the following traditions:  
Impressionism, Cubism, Abstract Expressionism, Dadaism.