Final Exam

1. If f is a positive continuous function, we define the area under the curve f between x = a and x = b as

$$\int_{a}^{b} f(t) dt = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_{i}^{*}) \Delta x_{i}.$$

Draw a picture which illustrates the right side of this formula. Explain how each part of the formula relates to the picture.

- 2. What is the definition of f'(x)? Use the definition to prove that (3x)' = 3.
- 3. State the Fundamental Theorem of Calculus (both parts). Explain in your own words what the FTC means.
- 4. Find the following integrals. (Don't forget to check your answers.)

$$\int x^2 - 4x \, dx$$
$$\int \cos(3x) \, dx$$
$$\int_0^{\pi/6} \sin t \, dt$$
$$\int \frac{x+1}{\sqrt{x^2+2x-1}} \, dx$$

- 5. Find the area of the football-shaped region which lies between the curves $y = x^2$ and $y = \sqrt{x}$.
- 6. The graph of f(t) is shown below. Graph $g(x) = \int_{-4}^{x} f(t) dt$. (Hint: You know that g(-4) = 0, and by the first part of the FTC, g'(x) = f(x).)

Extra Credit: In 1968, an American won the men's 200 meter run at the Olympics in Mexico City. For two points, name him. For two more points, explain why the U.S. Olympic Committee took away his gold medal.