Opportunity III Name:_____

This is your third chance to show me what you've learned this semester. No calculators are allowed. If you have any questions, please ask me or Beth. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. State the Mean Value Theorem. Be sure to include the hypotheses.

How is the Mean Value Theorem related to the rental car company using a GPS receiver to catch its clients speeding?

2. Show that the function $g(x) = x^{103} + 2x^{37} + x - 1$ has exactly one root. State any theorems you use.

3. Let $g(x) = x^3 - 3x^2 - 9x$. Find the maxima and minima (both local and absolute) of the function g on the interval $[-2, 4]$. Be sure to label each point (e.g. "local minimum") and justify your answer.

Let $h(x) = \sqrt{2}\sin x - \sqrt{2}\ x\cos x - \frac{1}{2}x^2$. Find the maxima and minima (both local and absolute) of the function h on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Be sure to label each point (e.g. "local minimum") and justify your answer.

4. The graph of $f'(x)$ is shown on the middle axes. Graph $f(x)$ on the top axes and $f''(x)$ on the bottom axes.

5. At a gravel pit, sand is falling onto a pile at a rate of 5 m^3/min. As the sand hits the pile, it falls down into a conical shape so that the height of the pile is always half of the radius. As the sand gets added, both the radius ($r(t)$) and the height ($h(t)$) grow. When $t = 1$ min., the height of the pile is 2 meters. (If you can't remember the formula for the volume of a cone, you can "buy" it for two points.)

Which is larger, $h'(2)$ or $h'(3)$? Why?

Calculate $h'(t)$ at the moment when the pile is 3 meters tall.

6. Find each limit. Justify your answer.

$$\lim_{x\to\infty} \frac{x^3 + 2x + 1}{5x^3 - 2x^2 + \pi}$$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1} + 2x}{x^3 + 3x - 1}$$

$$\lim_{x\to\infty} \frac{(\pi x - 1)(\pi x - 2)x}{(3x^2 - x)(2x + 1)}$$

7. Let $f(x) = \frac{1}{x+1} - \frac{1}{x}$.

Use facts about the asymptotes, the derivatives, the critical points and (if practical) the concavity of f to draw the graph of f. Use the space on the right to tabulate your results.

Extra Credit: Who penned the following lines? (One point for the author and one for the title.)
Turning and turning in the widening gyre
The falcon cannot hear the falconer
Things fall apart; the centre cannot hold;
Mere anarchy is loosed upon the world,

For an extra point, name the author who referenced this poem in a fictional work which examines the effects of European colonization in Africa.