

Opportunity II: The Derivative Strikes Back

Name: _____

Say the following out loud. "I am smart. I can do Calculus." Go ahead. Say it now.

This is your second chance to show me what you've learned so far this semester. No calculators are allowed. If you have any questions, please ask me. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. On the top set of axes is the graph of a function $g(x)$. Graph $g'(x)$ on the lower set of axes. Be sure to use the same scale (along the x -axis) for both graphs.

2. There are many different formulas which calculate the instantaneous change (a.k.a. the derivative) of a function. Is the following a correct formula?

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x) - f(x-h)}{h}$$

If you answer yes, explain why it correctly calculates the slope of f at the point x . If you answer no, give a correct formula and explain why it is correct. In either case, your explanation should explain each part of the formula.

3. Find $y'(x)$.

$$y = \cos(x^2)$$

$$y = x \sin(\pi x)$$

$$y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

$$y = \frac{x^2 - x - 2}{x^2 - 2x - 3} \text{ (simplify your answer)}$$

$$y = \tan(\sqrt{x} + x)$$

$$y = \frac{\sin(2x)}{\tan(2x)}$$

$$y = \frac{x^{1/3}}{x \sin(x)}$$

$$y^3 + xy - x^2 = 1,428,563.$$

4. Find $y''(x)$

$$y = \frac{1}{3} \left(\frac{\pi}{2} x \right)^2$$

$$y = \sqrt{x^2 + 1}$$

$$x^2 - y^2 = -1$$

5. Let $g(x) = \frac{1}{x^2}$.

Write down the definition of $g'(1)$.

Use this definition to prove that $g'(1) = -2$.

6. Find the following limits:

$$\lim_{x \rightarrow 0} \frac{\pi x}{\sin(2x)}$$

$$\lim_{z \rightarrow 0} \frac{z \cos(z)}{z^2 - z}$$

7. Choose one of the problems on this page to complete. Clearly mark which problem you want graded. If it is not clear which problem to grade, I will give you the lower of the two scores.

Let $g(x) = \csc(x)$. Prove that $g'(x) = -\csc(x) \cot(x)$.

Let $j(x) = h(g(x) + f^2(x))$. Write $j'(x)$ in terms of h , g , f , h' , g' , and f' .

Extra Credit: In recent times, five of our closer allies have been England, Israel, Pakistan, Mexico, and Saudi Arabia. Name the leader of each of those countries. Bonus points if you can circle those leaders who were elected.