

## Opportunity I

Here you go! This is your chance to show me what you've learned so far this semester. No calculators are allowed. If you have any questions, please ask me. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. The graph of the function  $f(x)$  is shown here. On the axes below, graph the following functions. For each, be sure to indicate several points on each axis. For one of the three, write a brief statement explaining why the graph looks the way it does.

a)  $h(x) = f(x + 1)$

b)  $g(x) = 2 + f(x - 1)$

c)  $k(x) = 2f(x) + 1$

2. In several sentences, explain what the following statement means:

$$\lim_{x \rightarrow 2} g(x) = 1.$$

Is the above statement true for the function  $g(x)$  shown here? Why or why not?

3. Use the various limit laws to show that

$$\lim_{x \rightarrow 1} \frac{x^2 + 3x + \sqrt{x}}{x^{1/3} - 2x^2} = -5.$$

Explicitly state which law you use in each step of your calculation.

4. Let

$$h(x) = \begin{cases} \frac{x^3-1}{x-1} & \text{if } x \neq 1 \\ ? & \text{if } x = 1. \end{cases}$$

What should you choose for the value of  $h(1)$  so that  $h$  is a continuous function for all  $x$ ? Why?

5. Calculate the following limits:

a)  $\lim_{x \rightarrow 2} \frac{x^2 + 3x + 1}{x - 1}$

b)  $\lim_{y \rightarrow \pi/3} \frac{2y^2}{(\tan y)^2}$

c)  $\lim_{z \rightarrow 2} \frac{z^2 + z - 6}{z^2 - z - 2}$

d)  $\lim_{w \rightarrow 1} \frac{w^2 + w - 2}{\sqrt{3} + w - 2}$

e)  $\lim_{s \rightarrow 1^+} \frac{s^2 + 2s + 1}{s - 1}$

6. Let  $g(x) = \sqrt{x + 2}$ .

Draw a rough sketch of  $g$ .

Find the tangent line to  $g$  at the point  $x = 2$ .

7. A model rocket is launched from on top of a picnic table by a curious 10 year-old kid. As she watches the rocket fly, she carefully calculates that for the first few seconds, the rocket's height is modeled by the equation

$$h(t) = -4t^2 + 10t + 1,$$

where time is measured in seconds and the height is measured in meters.

a) What is the average velocity of the rocket over the first second?

b) What is the instantaneous velocity of the rocket at  $t = 1$  second?

Bonus Question: After Mandarin Chinese, what are the next five most widely spoken languages in the world?