

Directions: You may use a calculator for this exam. If you have a TI-89 or TI-93, talk to me first. If you don't have a graphing calculator and would like to see the graph of a function, see me. In either case, you must justify every answer — simply saying that your graphing calculator showed you the answer will give you very little credit.

1. Derivatives!

Let $f(x) = \sin(x^2 + 3)$.

Write out the definition of $f'(3)$. Do not attempt to take the limit — just write out the definition.

Using the various rules we proved in class, find $f'(x)$.

Let $g(x) = \arctan(x)$.

Find $g(\sqrt{3})$.

Find $g'(x)$. You may *not* simply write down the answer.

2. More Derivatives! In each case, find y' .

$$y = 3x^2 + 2365e x^{19}$$

$$y = \ln(x^2 + \cos x)$$

$$ye^x = 1$$

$$y = \frac{\sin(x)}{\tan(x)}$$

$$y^4 + 3xy = \sin(y)$$

$$y = e^{x \cos x}$$

3. Integrals!

$$\int \frac{3}{1+x^2} dx$$

$$\int \frac{x+4}{x+1} dx$$

$$\int_0^2 e^{3x+4} dx$$

$$\int x^4 + 3x^2 + 6x + \frac{2}{x} dx$$

$$\int \cos(3x) dx$$

$$\int 4^x dx$$

- 4.** Cesium-137 is a byproduct of nuclear fission, and has a half-life of about 30 years. Unfortunately for us, it has many of the same chemical properties as potassium, and gets injected quite easily. Thus it's very important to keep Cs-137 confined until enough of it has decayed so as to be safe. Suppose a nuclear power plant removes 100 kg of Cs-137 from its reactor core. How many years will it be until less than 1 kg of Cs-137 is left?

- 5.** Integrate $\int_0^{\frac{\pi}{2}} \sin(2x) dx$. Explain how you are using the Fundamental Theorem of Calculus to compute the integral.

If $y = \int_1^x \frac{1}{\ln t} dt$, then what is y' ? Explain.

- 6.** Below are the graphs of three functions, labeled 1, 2, and 3. One of them is a function $f(x)$, one is the derivative of the first, namely $f'(x)$, and the third is the second derivative, $f''(x)$. Decide which graph is which, putting the appropriate numbers in the chart below, and explain why.

Function	Number
$f(x)$	
$f'(x)$	
$f''(x)$	

Explanation:

7. Write a question you think should have been on this exam; solve it. You will be graded both on how appropriate the problem is (e.g. “ $4 + 6 = ?$ ” will earn you zero points), and how well you solve it.