Exam 3                    Math 151                    Name: _____

**Directions:** OK, go ahead and use a calculator for this exam. If you have a TI-89 or TI-93, talk to me first. If you don't have a graphing calculator and would like to see the graph of a function, see me. In either case, you must give algebraic reasons for your answers.

**1.** Calculate the following integrals. In each case, your answer should be in the form of a single number.

$$\int_0^3 3x^2 + 2x - 1 \, dx \qquad\qquad\qquad\qquad \int_1^6 \sqrt{x+3}$$

$$\int_{-2}^2 x^3 - 2365e \ x \, dx \qquad\qquad\qquad\qquad \int_1^{14} \frac{1}{(2x-1)^{1/3}}$$

**2.** I am trying to make a small garden next to my house, using a straight 5 meter long fence. One side of the garden will be against the house, one side will be against the garage, and the hypotenuse will be formed by the fence. What is the maximum area I can enclose using my 5 meter fence? (If you get stuck in solving this problem, describe the steps you are trying to complete.)

**3.** Let $g(x) = x^3 - 3x^2 - 9x + 1$.

On what interval is $g$ decreasing? Give an algebraic reason.

What are the maximum and minimum values of $g$ on the interval $[0, 5]$?

Name all the inflection points of $g$. (Inflection points are where the graph of $g$ changes from being concave up to concave down or vice versa.)

**4.** Pick either the Extreme Value Theorem or the Mean Value Theorem and state it. Be sure to state the hypotheses of the theorem.

**5.** You savvy students should be prepared for this . . .

The formula for the integral of a function $f$ between $x = a$ and $x = b$ is:

$$\int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{k=1}^n \Delta x_k f(c_k).$$

Why does this formula give the area under the function $f$ between $a$ and $b$? Write a couple of paragraphs; be sure to include a description of what $\Delta x_k$ and $c_k$ are.

Write out the above expression when integrating the function $x^2 - x$ over the interval from $x = 1$ to $x = 2$, using $n$ equally spaced boxes and right end-points.

**6.** State both parts of the Fundamental Theorem of Calculus. For each part, give an explanation of what it means. (This explanation should be suitable for a calculus student who is having trouble understanding the Fundamental Theorem.)

**Extra Credit:** Prove the Pythagorean Theorem.