Opportunity

Instructions: Read each problem carefully. When in doubt, explain your answer thoroughly.

1. Find y':

$y = \cos(x\sqrt{x+1})$

$y^2 = \sin(xy)$

Calculate the following:

$\int_0^1 \sqrt{x} + x^{19} - \cos x \, dx$

$\int_0^{\pi/2} \sin(2x) \, dx$

Find $y(x)$:

$y' = \cos(2x)$, $y(\frac{\pi}{4}) = 1$

$y' = \frac{1}{x^2}$, $y'(-2) = 0$

2. If $f(x)$ is a positive, continuous function, then $\int_a^b f(x) \, dx$ gives the area under the curve $y = f(x)$ between $x = a$ and $x = b$. The precise definition is given in terms of the areas of boxes. Write a paragraph describing this definition.

State the Fundamental Theorem of Calculus (both parts), and briefly explain each part.

3. State the Mean Value Theorem. Be sure to note the hypotheses of the theorem.

Does the function

$$f(x) = \begin{cases} x^2 & x \geq 1 \\ x & x < 1 \end{cases}$$

on the interval $[0, 2]$ satisfy the hypotheses of the MVT? Give reasons for your answer.

4. In an isosceles trangle where the two equal sides have length 1 unit and the angle between those sides is θ, the area of the triangle is

$$A(\theta) = \sin(\frac{\theta}{2})\cos(\frac{\theta}{2}).$$

What is the largest such triangle? Justify your answer.

5. Let

$$g(x) = \frac{x^2}{x-1}.$$

What are the critical points of g?

Characterize all extrema of g. (i.e. Are they local or absolute extrema?)

Graph the function g.

6. Give the definition of the derivative.

Use this definition to calculate the derivative of $f(x) = x^2 + 2x$. (**Do not** use the power rule or the sum rule.)