Calculus 151 Name ______________

Opportunity 2

Instructions: Read each problem carefully. When in doubt, explain your answer thoroughly. If you have any questions, raise your hand or come up and ask me.

1. **Continuity:**

Give the formal definition of a function $f(x)$ being continuous at $x = c$.

Give an informal definition of a function $f(x)$ being continuous at $x = c$.

If $g(x) = \begin{cases} 2 \cdot \cos(x \cdot \frac{\pi}{6}) & x \leq 2 \\ x^3 - x^2 - 2 & x > 2 \end{cases}$, is g continuous at $x = 2$? Explain why or why not with a calculation or two **and** a sentence or two. (If you need to know the value of the cosine function at some point, I'll "sell" it to you for 2 points.)

2. Graph the function $f(x) = \frac{|x|}{x}$ and list its point(s) of discontinuity.

Let $j(x) = \begin{cases} \frac{\sin(2x)}{x} & x > 0 \\ x^2 + 2 & x \leq 0 \end{cases}$. Is j continuous at $x = 0$? Explain.

Calculate each limit or explain why it doesn't exist.

$$\lim_{z \to 0} \frac{z^2}{\tan^2(5z)}$$

$$\lim_{w \to 0} \frac{\cos^2(w)}{w^2}$$

$$\lim_{v \to 1} \frac{\sqrt{v} - 1}{v - 1}$$

3. State the Intermediate Value Theorem and draw a picture which illustrates it. (Use full sentences, please.)

State the Extreme Value Theorem and draw a picture which illustrates it. (Use full sentences, please.)

4. The definition of the derivative of a function $f(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

With a paragraph and a picture, explain why this formula gives the slope of the tangent line to the graph of $f(x)$ at the point x.

Using the definition of the derivative, prove the following two statements:

If $g(x) = \sin(x)$ then $g'(0) = 1$.

If $h(x) = \cos(x)$ then $g'(0) = 0$.

5. Prove that if $f(x) = \sqrt{x}$, then $f'(x) = \frac{1}{2\sqrt{x}}$.

Give an equation for the tangent line to $y = \sqrt{x}$ at $x = 4$.

6. Given the graph of $g(x)$ on the left, sketch the graph of $g'(x)$ on the right.