CALCULUS 151 NAME _____

Opportunity 1

Instructions: Here you go. It's your opportunity to show me how hard you've been working so far. Before you begin, say "I can do this." Out loud. Like you mean it. Then get started. (No calculators, no looking at other people's work, etc.) Don't forget to read the directions to each problem. Some ask for simple answers, some for graphs, some for verbal descriptions.

1. A couple of problems you've seen before...

What values of x satisfy $0 < |x - \frac{1}{2}| < 2$? Give you answer in interval notation *and* as a shaded region on a number line.

Give a detailed graph of $x^2 + y^2 - 2x - 4y + 1 = 0$ as well as a verbal description.

2. Domains, Ranges, Inverse Functions

Give the domain and range of the function

$$g(x) = \sqrt{|x| - 1}$$

Does $g(x)$ have an inverse function? Why or why not?

If $f(x) = (x + 3)^{-3} = \frac{1}{(x+3)^3}$, give a formula for $f^{-1}(x)$.

What is the range of $f^{-1}(x)$? Why?

3. **Calculating Limits:** Give values for the following quantities. When appropriate, show your work. If/when a limit doesn't exist, discuss why not. The first couple of questions refer to the graph on the right. The last couple do not.

$\lim_{x \to 0} f(x) =$

$\lim_{x \to -3^-} f(x) =$

$f(1) =$

$\lim_{x \to 0^+} f(x) =$

$\lim_{x \to 1} f(x) =$

$\lim_{x \to 0^-} f(x) =$

$$\lim_{x \to -1} \frac{x + 1}{x^2 - 5x - 6} =$$

$\lim_{z \to 0} \frac{\cos(z)}{z^2 + \pi^2} =$

4. You knew they had to come sooner or later....**ϵ's and δ's**

Give the formal definition of $\lim_{x \to c} f(x) = L$.

Give a more informal, intuitive definition (as if you were explaining what it meant to a friend) of the same statement. Include a picture if it helps.

5. Give an $\epsilon - \delta$ proof that $\lim_{x \to -2} 2x + 5 = 1$. Do this in two steps:

Figure out what you should choose for δ.

Proof that your choice of δ works. For this part, write in full sentences - with proper punctuation.