Practice Exam 2

(1) State the limit definition of the derivative.

$$f'(x) = ...$$

(2) State the following differentiation rules: Product Rule, Quotient Rule, Power Rule, and Chain Rule.

(3) Suppose you have two functions f and g whose composition satisfies

$$f\left(g(x)\right) = x^2.$$

If we know that $g(0) = 1$ and that $g'(0) = \pi$, what do we know about $f'(1)$? What do we know about $f'(0)$?

(4) Find the equation of the line tangent to $y = \tan x + \cot x$ at the point $(\pi/4, 2)$.

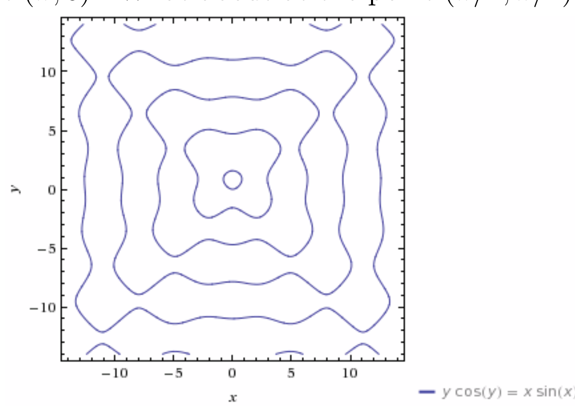
(5) Given $f(x) = \sqrt{x}$ and $g(x) = \sin x$ differentiate the following functions:

$$(f/g), \qquad f \cdot g, \qquad \frac{f+g^2}{f-g}, \qquad f(g(x)), \qquad g(f(x)).$$

(6) The curve associated to the equation

$$y \cos y - x \sin x = 0$$

is shown below. What is the slope of the line tangent to this curve at the point $(\pi, 0)$? What about at the point $(\pi/4, \pi/4)$?



(7) Suppose we have a function $f(x)$ defined on some closed interval $[a,b]$ that is differentiable on the open interval (a,b) and has a negative derivative: $f'(x) < 0$ for every x in (a,b). At what point, if any, does $f(x)$ obtain its global maximum? What about its global minimum? Explain your answer(s).

(8) Some student worked out the following derivative:

$$f(t) = \sqrt{1+\cos t + t^{-1}}.$$
$$f'(t) = \frac{1}{2}\left(1+\cos t + t^{-1}\right)^{-1/2}$$

Did this student differentiate correctly? If so, explain how it is you know this. If not, correct their work.

(9) What is the 1000th derivative of $y = \cos x$?

(10) What is the maximum value of $g(t) = (t^2+t+1)/(t+2)$ on the interval $[-1,1]$?

(11) Compute the first four derivatives of $p(x) = x^4 + x^3 + x^2 + x + 1$. What is the 5th derivative of $p(t)$? What is the 6th derivative? The 7th? The 10-millionth?

(12) Hi! Do some suggested problems, too!

Answers

(1) It involves limits of quantities like h going to zero ...

(2) Here they are! But which is which???????!!?!?

$$\left(\frac{f}{g}\right)' = \frac{f'\,g - f\,g'}{g^2}$$

$$(f \cdot g)' = f'g + g'f$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$(x^n)' = nx^{n-1}$$

(3) Using the chain rule, one finds that $f'(1) = 0$. However, it is impossible to determine $f'(0)$ from the information given.

(4) The equation is $y = 2$.

(5) In order, the derivatives are

$$\frac{(1/2)x^{-1/2}\,\sin x - \sqrt{x}\cos x}{\sin^2 x}, \qquad \frac{1}{2}x^{-1/2}\,\sin x + x^{1/2}\,\cos x,$$

$$\frac{((1/2)x^{-1/2} + 2\sin x\cos x)(\sqrt{x} - \sin x) - (\sqrt{x} + \sin^2 x)((1/2)x^{-1/2} - \cos x)}{(\sqrt{x} - \sin x)^2}$$

$$\frac{1}{2}(\sin x)^{-1/2}\,\cos x, \qquad \cos(\sqrt{x}) \cdot \frac{1}{2}x^{-1/2}$$

(6) We did this in class.

(7) We did this in class, too.

(8) This answer is incomplete. The full answer is

$$f'(t) = \frac{1}{2}\left(1 + \cos t + t^{-1}\right)^{-1/2} \cdot \left(-\sin t - t^{-2}\right).$$

(9) $y^{(1000)}(x) = \cos x$.

(10) We did this one in class. The critical points are $-2 \pm \sqrt{3}$, but only one of these points is in this interval. The maximum value is found by comparing the function evaluated at this critical point with the function evaluated at the end points.

(11) The first four derivatives are $p'(x) = 4x^3 + 3x^2 + 2x + 1, p''(x) = 12x^2+6x+2, p'''(x) = 24x+6$, and $p^{(4)}(x) = 24$. All higher derivatives are the constant 0.

(12) Okay!