Practice Exam

(1) Sketch a graph of $y = \dfrac{1+2x}{x^2+1}$. Be sure to indicate local maxima, local minima, inflection points, and asymptotes.

(2) Sketch a graph of $y = f(x)$ where $f(x)$ is *any* function satisfying the following properties:

$$\lim_{x\to\pi} f(x) = \infty \qquad \lim_{x\to\infty} f(x) = -\infty \qquad \lim_{x\to-\infty} f(x) = -10$$

$$f'(0) = 0 \qquad f'(x) > 0 \text{ on } (-\infty, 0) \qquad f'(x) < 0 \text{ on } (0, \infty)$$

$$f''(x) > 0 \text{ on } (-\infty, -1) \qquad f''(x) < 0 \text{ on } (-1, \infty)$$

(3) Use limits of Riemann Sums to determine the (signed) area under the graph of $y = x + x^2$ along the interval $[4, 5]$. (Hint: you will need to use summation formulas.)

(4) Of all right triangles with perimeter 10, which one(s) (if any) enclose the most amount of area?

(5) My friend Alex thinks that the limit below can be viewed as a formula for the area under some curve:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left| i\frac{2\pi}{n} - \pi \right| \frac{2\pi}{n} = \int_a^b f(x)\,dx.$$

Determine a, b, and $f(x)$. Also, which endpoints were used in setting up this problem? Finally, using any methods you desire, compute the area.

(6) Hi! What does the Mean Value Theorem say? What does Rolle's Theorem say? Draw pictures of both theorems.

(7) My friend Emek has cooked up a nifty function, $H(t) = \dfrac{\sin(t)}{4 + \sin(t)}$, which she claims models the happiness experienced by calculus students at time t. For her function, $t = 0$ corresponds to the start of the semester, and $t = 12$ corresponds to the current time (are we in the 12th week?). Sketch a graph of $y = H(t)$. What so far has been the happiest moment? When has the saddest moment occurred? Locate all inflection points and describe what they mean.

(8) Explain how the expression

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} f(a + i\triangle x)\triangle x$$

represents (signed) area. Be sure to explain the meaning of each expression!

(9) Set up a limit of Riemann sums that equals the area represented by

$$\int_{-1}^{1} \sin\left(x^3\right)\, dx.$$

While we do *not* have summation formulas that let us evaluate your limit, some of my friends are convinced that they know the answer. In particular, Ivan is convinced that this (signed) area adds up to zero, while Dave claims the area has to be some positive value with Susan championing the possibility that the area is negative. Who is correct? Explain your reasoning.