The Final (Countdown) Exam

Instructions: You may use proven results from the *class* textbook, you may consult your notes, and you may use assumptions and facts about sets learned in F.O.M. and discussed in this class. No other resources are allowed. If a theorem or result in the textbook is needed for your solution and this result has not yet been proved, you may cite the result (but you will not receive full credit unless you include a proof for the cited item; material from the additional sections can be cited). This exam is due at the start of our final exam.

The so-called "quickie problems" do not require rigorous explanation. Pictures and logical sketches and ideas suffice.

Quickies

Quickie Problem 1.

(a) Compute the fundamental group of S^1 at the base point $x_0 = (1, 0)$.

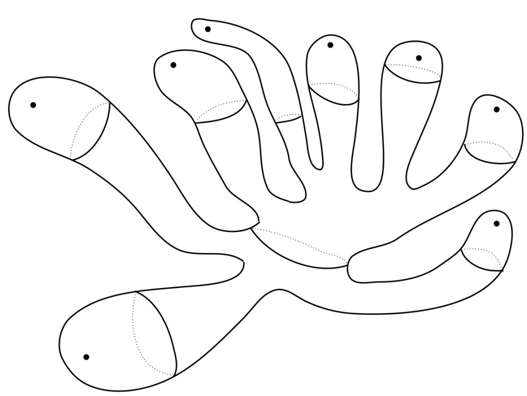
(b) Compute the fundamental group of S^1 at the base point $x_0 = (0, 1)$.

(c) Compute the fundamental group of $S^1 \vee S^1$ (at the base point x_0 = wedge point).

(d) Compute the first *homology* group of $S^1 \vee S^1$.

(e) Let X be the identification space obtained by glueing in a B^2 along the boundary of *one* of the circles in the space $S^1 \vee S^1$. Compute the fundamental group of X (at any base point you like).

Quickie Problem 2.
Let X denote the subspace of $\mathbb{E}^3$ (a.k.a. Cthulu monster) shown below.



The dots designate particular points, and the loops shown on the figure are there only to help visualize it as a two-dimensional object sitting in three-dimensional space.

(a) What is the genus of X?

(b) Compute the fundamental group of X.

(c) Let x_1 denote any one of the indicated points. Compute the fundamental group of $X \backslash \{x_1\}$.

(d) Compute the fundamental group of $X \backslash \{\text{all indicated points}\}$.

Quickie Problem 3.
S^1 yourself from the image below.

Quickie Problem 4.
Pasquale claims that every covering space of $S^1 \times S^1$ is regular a.k.a. normal. Is he correct? Explain your answer.

Proofy Problems

Problem 1. Prove your favorite theorem from this class. Why is it your favorite theorem?